

EXPERIMENTAL ANALYSIS TO DETERMINE  
THE NUMBER OF REQUIRED CALIBRATION LEVELS  
AND SAMPLES AT EACH LEVEL  
FOR THE FM/FM TELEMETRY SYSTEM

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FACILITY FORM 602

N67-28835

(ACCESSION NUMBER)

103

(PAGES)

CR-84831

(NASA CR OR TMX OR AD NUMBER)

(THRU)

(CODE)

(CATEGORY)

August, 1966

TECHNICAL REPORT NUMBER 16

**SYSTEMS ENGINEERING GROUP**

**BUREAU OF ENGINEERING RESEARCH**

**UNIVERSITY OF ALABAMA UNIVERSITY, ALABAMA**



SQT-44370

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TECHNICAL REPORT NUMBER 16

prepared for

National Aeronautics and Space Administration  
Marshall Space Flight Center  
Huntsville, Alabama

under  
CONTRACT NUMBER NAS 8-20172

Systems Engineering Group  
Bureau of Engineering Research  
University of Alabama

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## ABSTRACT

This is the sixteenth of a series of technical reports concerned with the Telemetry Systems on the Saturn Vehicle.

As an extension of Technical Report Number 9, a methodology is developed to determine the mathematical model which relates input to output of a telemetry system. Since it is infeasible to develop the model under the actual environmental conditions, a simulation model reflecting the linearity characteristics of the true system is employed. Three levels of random noise are recognized as existing in the system. These three levels are respectively considered for the second, third, and fourth degree coefficients of the simulated polynomial relation.

The regression analysis, with orthogonal polynomials technique, is used to find the curve providing the adequate fit. To determine the optimal calibration levels and sample size at each level, analysis of variance in conjunction with Duncan's Multiple Range Test is employed.

Under the simulated experimental conditions, two calibration steps with fifteen samples at each step is significantly better than other techniques.

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## CHAPTER I

### INTRODUCTION

#### Summary of Past Experiments

Previous experimentation has indicated that the FM/FM telemetry system may be moderately non-linear (1,2).<sup>1</sup> To be more specific, the mathematical model used at present in the data reduction process for the system relationship is a fourth degree polynomial. The method used to relate functionally the output and input of the FM/FM telemetry system is Lagrangian Interpolation. This method, as shown in (1), possesses greater reconstruction errors than those of least squares regression analysis for recognizing the random errors existing in the system.

Referring again to (1), a methodology of least squares regression analysis coupled with simulation technique was recommended to estimate the relationship present in the system. This report also presented the results of an investigation of the number of calibration levels, N, and the number of sample size, M, required to fit an appropriate curve to the output of a telemetry system.

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<sup>1</sup>Numbers in parentheses throughout the thesis indicate the reference as listed in the List of Reference.

One of the hypotheses made in that investigation was that a calibration sequence of more than five levels might be needed to accurately determine the functional relationship between the input and the output of the system. Hence the levels chosen to be investigated were 5, 7, 9, and 11.

A second hypothesis was that the number of sample points needed to accurately determine the output at each level was either five or ten.

Earlier investigations indicated that there was no bias in the output of the system. These investigations also showed that normally distributed random noise existed. The best estimate for this noise was that it was normally distributed with mean zero and some standard deviation  $\sigma_e$ , where

$\sigma_e$  is about 1% to 1.5% of full range. Hence in Technical Report Number 9, the noise variables <sup>2</sup> considered were 0.5%, 1.0%, 1.5% and 2.0%.

The conclusion reported in Technical Report Number 9 was that it would be desirable to use five calibration levels with at least ten samples at each level. This report also concluded that it would be desirable to investigate the feasibility of using less than five calibration levels, and to consider more than ten samples at each level.

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<sup>2</sup>In statistics language, this means that noise can be considered as a random variable.

### The Present Problem

This report is an extension of Technical Report Number 9. A simulation of the type described in that report has been conducted for 2, 3, 4, and 5 calibration levels with 5, 10, and 15 samples at each level. The purpose of selecting these values is to determine whether or not fewer calibration levels with an appropriate sample size are adequate to describe the relationship present in the system. The meaning of 'adequate' here is that it is not statistically significantly different from the 'best' at a certain confidence level, viz. 95%. If indeed a shorter calibration sequence is found to be adequate, the costs and efforts in future calibration procedures may be reduced.

In this report the following questions will be investigated:

1. Are present standard calibration levels ( $N=5$ ) optimal? If not, which are the optimal ones;  $N=4$ , 3 or 2 levels?
2. Is sample size at each level,  $M = 10$ , large enough?
3. Do the coefficients of the output function which is assumed a fourth degree polynomial, within certain specified ranges, significantly affect the selection of the optimal  $M$  and  $N$ ?

In Chapter II, the methodology for simulating the system outputs and for finding the adequate curves will be developed. The technique of orthogonal polynomial will be used in regression analysis. An illustration will be presented to help familiarize the reader with the methodology. Chapter III is devoted to estimating the parameters under investigation by techniques of statistical analysis. Also presented in Chapter III is a summary of the results and the pertinent interpretations. Chapter IV summarizes the conclusions from the preceding chapters.

## CHAPTER II

### SIMULATION MODEL

#### Simulation Data

In the determination of an optimal number of calibration levels and an optimal sample size at each level, it would be desirable to perform an experiment on the actual system. However, this is not feasible for several reasons (1); 1. Under actual environmental conditions, the precision could not be varied or replicated accurately enough to determine the two parameters under study. 2. The considerable amount of experimentation which would be required for meaningful estimates of the parameters would be infeasible. Therefore, the data, instead of being obtained from actual experimentation, will be generated through the technique of simulation. Using values of precision which reflect the actual environment of the system, data can be generated for varying conditions. Simulation also allows for replications of a given set of conditions. Moreover, a considerable amount of data may be generated rapidly by means of an electronic digital computer.

In the simulation of the telemetry system, an actual

relationship of the system output  $F(X)$  to input  $X$  will be assumed as portrayed in equation (1).

$$F(X) = A_0 + A_1X + A_2X^2 + A_3X^3 + A_4X^4 + e_i \quad (1)$$

where,  $A_k$  = coefficients of the function,  $k = 0, 1, 2, 3, 4$ .

$e_i$  = random noise, normally, independently distributed with mean zero and variance  $\sigma_e^2$ .

The reasons for assuming the system relationship as a polynomial of degree four are (1):

1. Previous studies using 21 levels for all channels in an actual telemetry system indicated that the highest significant degree was probably no higher than the fourth degree polynomial and usually less.
2. For the present standard calibration sequence of five steps, the highest degree relationship which can be estimated is a fourth degree.

The specific factors to be considered in this experiment may be summarized as follows:

$N$  = calibration level, i.e., four different levels<sup>5</sup> namely 2, 3, 4, and 5.

$M$  = sample size at each calibration level, i.e., three different levels namely 5, 10, and 15.

---

<sup>3</sup> For  $N = 2$ , samples are taken at 0 and 5 volts (or 0% and 100%). For  $N = 3$ , samples are taken at 0.00, 2.50, and 5.00 volts (or 0%, 50% and 100%). For  $N = 4$ , samples are taken at 0.00, 1.67, 3.33 and 5.00 volts (or 0%, 33%, 67% and 100%). For  $N = 5$ , samples are taken at 0.00, 1.25, 2.50, 3.75, and 5.00 volts (or 0%, 25%, 50%, 75% and 100%).

$$A_0 = 25^4$$

$$A_1 = 200^4$$

$$A_2 = \text{three different levels: } 0.00, 0.75 \text{ and } 1.50^5$$

$$A_3 = \text{three different levels: } 0.00, 0.30 \text{ and } 0.60^5$$

$$A_4 = \text{three different levels: } 0.00, 0.05 \text{ and } 0.10^5$$

$$e_i = \text{three different levels: } 0.2\%, 0.6\% \text{ and } 1.0\% \text{ of full range}^5$$

Replications = 5.

### Illustration

In order to provide the reader with a better understanding of the practical aspects of the simulation technique and the curve fitting procedure, an example will be presented in this section.

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<sup>4</sup>These values were selected arbitrarily for the linear terms of the output function depending on the range within which the digitized (dimensionless) output may vary.

<sup>5</sup>The upper limits were so chosen that their effects on non-linearity may not significantly affect the linearity of the actual system. Once the upper limits were set, the other values were placed at equal intervals.

Suppose a telemetry system existed for  $A_0 = 25$ ,  $A_1 = 200$ ,  $A_2 = 0.75$ ,  $A_3 = 0.30$  and  $A_4 = 0.05$ , then the true output may be expressed symbolically by:

$$f(X) = 25 + 200 X + 0.75 X^2 + 0.30 X^3 + 0.05 X^4 \quad (2)$$

### Simulation of the Observed Output

As mentioned before, the observed output is not coincident with the true output due to the existence of random noise,  $e_i$ . Recognizing the random noise, the observed output may be stated as:

$$F(X) = 25 + 200X + 0.75X^2 + 0.30X^3 + 0.05X^4 + e_i \quad (3)$$

or  $F(X) = f(X) + e_i$ .

It was also mentioned that the error  $e_i$  was normally distributed with zero mean and variance  $\sigma_e^2$ . In previous reports, the error was commonly expressed in terms of precision  $\sigma_p$ . The relationship between  $\sigma_e$  and  $\sigma_p$  is:

$$\sigma_p = 100 \frac{\sigma_e}{R} \quad (4)$$

where,  $R$  is actual full range of output. If we let  $X_{\max}$  and  $X_{\min}$  denote the maximum and minimum inputs of the system respectively, then  $R$  is defined by  $R = f(X_{\max} - X_{\min})$ . Substituting  $X_{\max} = 5.0$  and  $X_{\min} = 0.0$  in expression (2), the value of the full range may be obtained as 1097.50.

Suppose the given precision of the system  $\sigma_p = 0.6$ ,

then from equation (4):

$$\sigma_e = \frac{\sigma_p}{100} \cdot (R) = \frac{0.6}{100} (1097.50) = 6.585$$

Once the standard deviation of the random error  $\sigma_e$  is determined, the nature of normal random numbers is used to simulate the random error  $e_i$ ,

$$e_i = \sigma_e \text{ times the random normal number} \quad (5)$$

A random normal number is a random sample value from a normal distribution with mean equal to zero and standard deviation of one. The value may be found in a specific table or generated directly by a computer as it was done in Technical Report Number 9. Each time a random error is generated, a new random normal number should be selected.

A set of observed output can then be simulated by substituting a set of random errors into equation (3).

Suppose a sample of ten is desirable at  $X = 0.0$  volt, then a set of ten random normal numbers should be generated. From the values of this set, the corresponding ten random samples can be computed by expression (5). Substituting these values in equation (3), the observed output are obtained as listed in column 2 of Table 1.

TABLE 1

AN EXAMPLE OF SIMULATED OBSERVED OUTPUT

	Calibration Levels in Volts				
	0.00	1.25	2.50	3.75	5.00
Sample Size  M = 10	24.039	266.613	522.322	812.565	1112.830
	21.273	271.243	532.871	812.565	1111.058
	13.819	284.216	532.871	812.348	1106.896
	17.223	278.276	543.183	826.223	1113.040
	18.145	284.459	535.261	809.925	1113.883
	21.589	279.613	530.408	811.525	1107.581
	35.668	264.112	525.674	811.534	1120.837
	30.551	276.083	528.992	803.801	1117.702
	31.894	297.415	538.817	810.219	1108.721
	28.846	275.261	536.374	812.816	1113.455
$\bar{y}$	24.305	275.929	532.591	812.126	1112.600
$\sigma_e^2$	51.211	46.898	38.204	30.871	19.814

The values of means  $\bar{Y}$  and variance  $\sigma_e^2$  for each calibration level may be illustrated by:

$$\bar{Y} = \frac{\sum_{i=1}^{i=10} Y_i}{M} = 24.305$$

$$\sigma_e^2 = \frac{1}{M-1} \left( \sum_{i=1}^M Y_i^2 - M \bar{Y}^2 \right) = 51.211$$

### Orthogonal Polynomials

One method to fit the empirical data is the method of least squares. The method states that the best representative curve is that for which the sum of the squares of the errors is a minimum. In the determination of the best curve relating to the simulated observed data, the method of least squares is used to minimize the squared deviations between the observed and the fitted curves.

To determine the coefficients of the fitted curve, traditionally a system of normal equations is set up. These equations are solved for the unknown coefficients, e.g.,  $a_0$  and  $a_1$ , for a linear polynomial. In general, the values of  $a_0$  and  $a_1$ , for a quadratic model will not coincide with those for a linear model. Hence, whenever a higher degree polynomial is fitted, the values of the coefficients should be recomputed and the values of the test statistics in regression analysis will be changed. Moreover, the traditional me-

thod is found difficult in practice for solving the normal equations for a curvilinear model and still maintain sufficient accuracy.

The technique of orthogonal polynomials is thus used to eliminate these disadvantages. In order to make use of this technique, the output function is rewritten as (6),

$$F(P) = b_0P_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 \quad (6)$$

where the P's are orthogonal polynomials and the b's are coefficients. The values of orthogonal polynomials depend on the method of coding the inputs X's and number of calibration levels used. In the previous example the inputs are calibrated at 0.00, 1.25, 2.50, 3.75 and 5.00 volts and coded as shown in equation (7).

$$Z = \frac{X - 2.50}{1.25} \quad (7)$$

Then the relations between the coded input Z and the orthogonal polynomials through the fourth degree are:

$$P_0 = 1 \quad (8)$$

$$P_1 = Z \quad (9)$$

$$P_2 = Z^2 - (N^2 - 1)/12 \quad (10)$$

$$P_3 = Z^3 - (3N^2 - 7)Z/20 \quad (11)$$

$$P_4 = Z^4 - (3N^2 - 13)Z^2/14 + 3(N^2 - 1)(N^2 - 9)/560 \quad (12)$$

Referring to equations (7) through (12), the necessary computations for the P's are exhibited in Table 2. The b's can also be obtained by substituting into equations (13) through (17).

$$b_0 = \frac{\sum_{i=1}^5 P_0 \bar{Y}_i}{\sum_{i=1}^5 P_0^2} = \frac{2757.551}{5} = 551.510 \quad (13)$$

$$b_1 = \frac{\sum_{i=1}^5 P_1 \bar{Y}_i}{\sum_{i=1}^5 P_1^2} = \frac{2712.787}{10} = 271.279 \quad (14)$$

$$b_2 = \frac{\sum_{i=1}^5 P_2 \bar{Y}_i}{\sum_{i=1}^5 P_2^2} = \frac{120.573}{14} = 8.612 \quad (15)$$

$$b_3 = \frac{\sum_{i=1}^5 P_3 \bar{Y}_i}{\sum_{i=1}^5 P_3^2} = \frac{15.901}{10} = 1.590 \quad (16)$$

$$b_4 = \frac{\sum_{i=1}^5 P_4 \bar{Y}_i}{\sum_{i=1}^5 P_4^2} = \frac{-19.769}{70} = -0.282 \quad (17)$$

---

<sup>6</sup>The properties of orthogonal polynomial and their rationales can be found in Technical Report Number 9.

TABLE 2

## COMPUTATIONS FOR ORTHOGONAL POLYNOMIALS

(Left)

X	Z	$\bar{Y}_i$	Orthogonal Polynomials										$P_0 \bar{Y}_i$
			$P_0$	$P_0^2$	$P_1$	$P_1^2$	$P_2$	$P_2^2$	$P_3$	$P_3^2$	$P_4$	$P_4^2$	
0.00	-2	24.305	1	1	-2	4	2	4	-1	1	1	1	24.305
1.25	-1	275.929	1	1	-1	1	-1	1	2	4	-4	16	275.929
2.50	0	532.591	1	1	0	0	-2	4	0	0	6	36	532.591
3.75	1	812.126	1	1	1	1	-1	1	-2	4	-4	16	812.126
5.00	2	1112.600	1	1	2	4	2	4	1	1	1	1	1112.600
Sum			5		10		14		10		70		2757.551

(Right)

X	$P_1 \bar{Y}_i$	$P_2 \bar{Y}_i$	$P_3 \bar{Y}_i$	$P_4 \bar{Y}_i$	$\bar{Y}_i^2$
0.00	-48.610	48.610	-24.305	24.305	590.733
1.25	-275.929	-275.929	551.858	-1103.716	76136.813
2.50	0	-1065.182	0	3195.546	283653.173
3.75	812.126	-812.126	-1624.252	-3248.504	659548.640
5.00	2225.200	2225.200	1112.600	1112.600	1237878.760
Sum	2712.787	120.573	15.901	-19.769	2257808.119

These values are unchanged, once obtained, for either linear, quadratic, cubic, quartic or quintic polynomials.

### Fitting an Adequate Curve to the Simulated Data

Recall that the actual relationship of the system was assumed expressed by a polynomial. The observed data may also be fitted by a polynomial. Beginning with a linear model the expression  $Y = 551.510P_0 + 271.279 P_1$  is used to fit the data. To test whether this linear model provides an adequate fit, the technique of regression analysis will be employed.

The computations for obtaining the necessary sum of squares are as follows: Let  $SS_{b_0}$ ,  $SS_{b_1}$ ,  $SS_{M_1}$ ,  $SS_e$  and  $SS_T$  denote sum of squares for  $b_0$ ,  $b_1$ , means of the linear model, error and total respectively, then

$$SS_{b_0} = \frac{M (\sum P_0 \bar{Y}_i)^2}{P_0^2} = \frac{10(2757.551)^2}{5} = 15208175.036$$

$$SS_{b_1} = \frac{M (\sum P_1 \bar{Y}_i)^2}{P_1^2} = \frac{10(2712.787)^2}{10} = 7359213.308$$

$$\begin{aligned} SS_{M_1} &= M \sum \bar{Y}_i^2 - SS_{b_0} - SS_{b_1} \\ &= 22578081.190 - 15208175.036 - 7359213.308 \\ &= 10692.846 \end{aligned}$$

$$\begin{aligned}
 SS_e &= \sum_1^5 (M - 1) \sigma_e^2 \\
 &= 9(51.211 + 46.898 + 38.204 + 30.871 + 19.814) \\
 &= 1682.982 \\
 SS_T &= SS_{b_1} + SS_{M_1} + SS_e = 7371589.136
 \end{aligned}$$

The analysis of variance is presented in Table 3, where the confidence level is 95%. Since the variance ratio of mean to error is greater than the significant level, the hypothesis that the chance variation of the linear model is less than 5% is rejected. It is then concluded that the linear model will introduce significant inaccuracy in fitting the observed data.

TABLE 3

## THE ANALYSIS OF VARIANCE FOR THE LINEAR MODEL

Source of Variance	Sum of Squares	df	Mean Squares	Variance Ratio	Expected F Value at 95% level
Linear	7359213.308	1	7359213.308		
Mean	10692.846	3	3564.282	95.302*	2.815
Error	1682.982	45	37.400		
Total	7371589.136	49			

\*Significant at 95% confidence level.

Since the linear model failed to properly fit the data, a quadratic model will be used next. In the curve-fitting recursive process, the lower degree coefficients of the orthogonal polynomial are unchanged. In the previous example,  $b_0$  and  $b_1$  will remain 551.510 and 271.279 respectively for the second degree polynomial. The second degree coefficient  $b_2$  however, will be obtained by expression (15). Therefore the new fitting curve becomes  $Y = 551.510 P_0 + 271.279 P_1 + 8.612 P_2$ .

Again, the sum of squares for  $b_2$  and the new mean can be computed:

$$SS_{b_2} = \frac{M (\sum P_2 Y_i)^2}{P_2^2} = 10384.178$$

$$SS_{M_2} = M \sum Y_2^2 - SS_{b_0} - SS_{b_1} - SS_{b_2} = 308.668.$$

An analysis of variance for this quadratic model was performed and the results are shown in Table 4. The variance ratio is still significant at the 95 % confidence level, and thus the hypothesis that the errors of this quadratic model are due to chance is still rejected. In other words, the quadratic model is not adequate to describe the set of observed data.

TABLE 4

## THE ANALYSIS OF VARIANCE FOR THE QUADRATIC MODEL

Source of Variance	Sum of Squares	df	Mean Squares	Variance Ratio	Expected F Value at 95% Level
Linear	7359213.308	1	7359213.308	4.127*	3.205
Quadratic	10384.178	1	10384.178		
Mean	308.668	2	154.334		
Error	1682.982	45	37.400		
Total	7371589.136	49			

\*Significant at 95% confidence level.

Then a cubic model, equation (18), was used to fit the data:

$$Y = 551.510 P_0 + 271.279 P_1 + 8.612 P_2 + 1.590 P_3 \quad (18)$$

The value of  $b_3$  is computed in equation (16) where the sum of squares for  $b_3$  and the cubic mean are given by:

$$SS_{b_3} = \frac{M(\sum P_3 \bar{Y}_i)^2}{P_3^2} = \frac{10(15.901)^2}{10} = 252.842$$

$$SS_{M_3} = MP\bar{Y}_i - SS_{b_0} - SS_{b_1} - SS_{b_2} - SS_{b_3} = 55.826$$

The variance ratio for the cubic model indicated that it is not significant at the 95% confidence level. Hence the cubic model as related in equation (18) is determined to be the true relation of the system.

TABLE 5  
THE ANALYSIS OF VARIANCE FOR THE CUBIC MODEL

Source of Variance	Sum of Squares	df	Mean Squares	Variance Ratio	Expected F Value at 95% level
Linear	7359213.308	1	7359213.308	1.493	4.055
Quadratic	10384.178	1	10384.178		
Cubic	252.842	1	252.842		
Mean	55.826	1	55.826		
Error	1682.982	45	37.400		
Total	7371589.136	49			

A conceptual plot of the fitted curve versus the actual curve of the system is shown in Figure 1, and a general layout of analysis of variance is displayed in Table 6.

When the appropriate degree of the polynomial is determined, it may be desirable to estimate the accuracy and precision for this model. The accuracy and precision may be

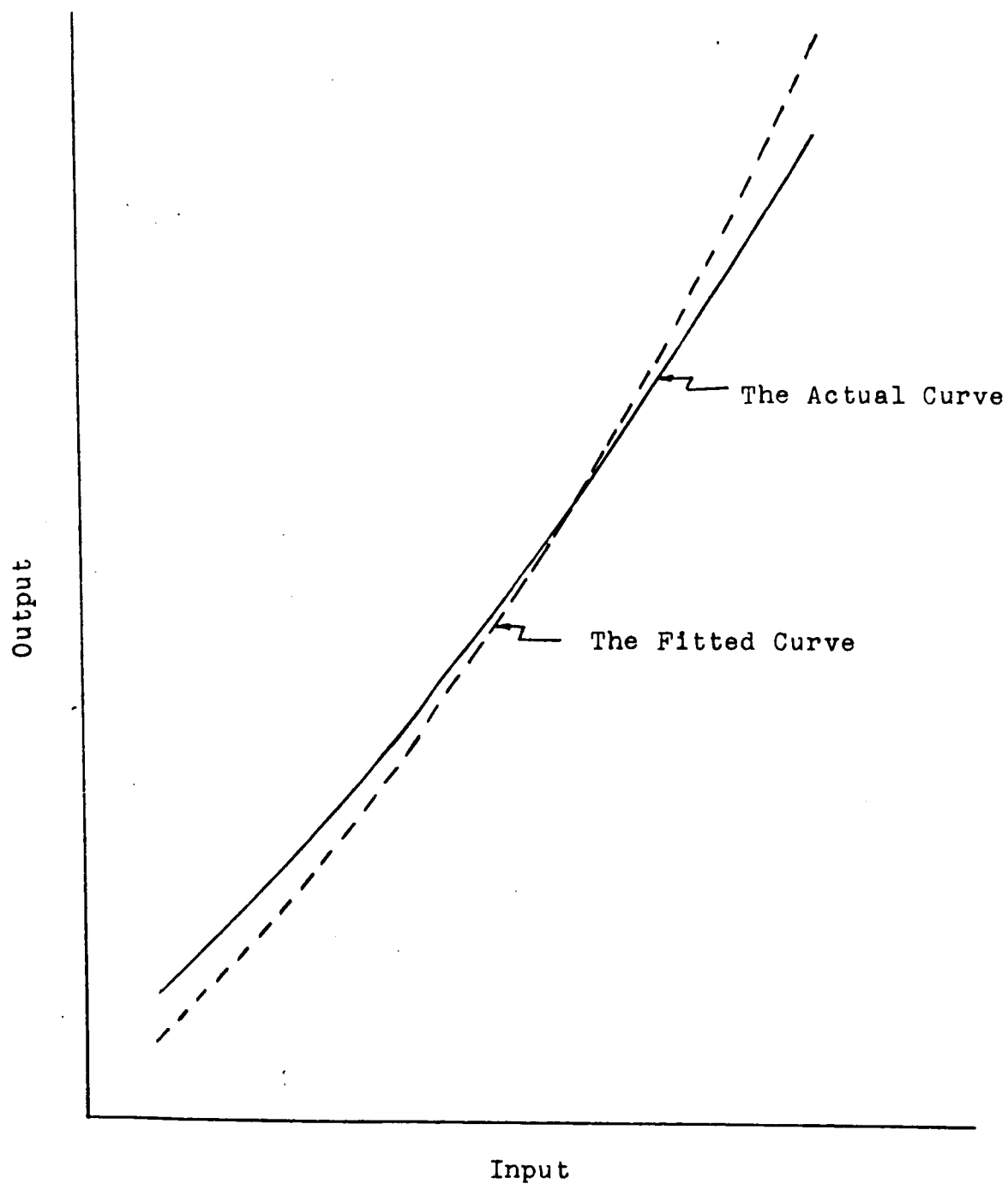


FIGURE 1 A CONCEPTUAL PLOT OF THE FITTED CURVE  
VERSUS THE ACTUAL CURVE OF THE SYSTEM

TABLE 6  
A GENERAL LAYOUT OF ANALYSIS OF VARIANCE

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Squares	Variance Ratio
$b_1$	$SS_{b_1}$	1	$S_{b_1}^2$	$S_{b_1}^2 / S_M^2$
$b_2$	$SS_{b_2}$	1	$S_{b_2}^2$	$S_{b_2}^2 / S_M^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$b_k$	$SS_{b_k}$	1	$S_{b_k}^2$	$S_{b_k}^2 / S_M^2$
Mean	$\sum \sum (\bar{Y}_i - Y_i)^2$	$N-k-1$	$S_M^2$	
Error	$\sum \sum (Y_{ij} - \bar{Y}_i)^2$	$N(M-1)$	$S_e^2$	
Total	$\sum \sum (Y_{ij} - Y_i)^2$	$NM-1$	$S_T^2$	

estimated by the mean variance for the mean and the mean variance for the error respectively. Computed from Table 1, the average range of the output is 1088.296. The estimate of the precision is:

$$\sigma_p = 100 \sqrt{s_e^2} / \bar{R} = 100 \sqrt{37.40} / 1088.296 = 0.562\%$$

The estimate of the accuracy is:

$$\begin{aligned} \sigma_a &= 100 \sqrt{(s_M^2 - s_e^2) / M} / \bar{R} \\ &= 100 \sqrt{(55.826 - 37.400) / 10} / 1088.296 = 0.125\% \end{aligned}$$

Note that the values of the estimates are in percent of full range.

### Criteria For the Selection of Parameters

As a measure of the effectiveness of the fitted (regression) curve  $Y$  to the data  $f(X)$ , the standard error for these two curves are computed. By sampling a large amount of equispaced input values, say 100 in this experimentation, we can obtain a set of corresponding outputs for these two curves. The standard error of the fitted curve is then defined by equation (19).

$$\sigma_{y.x} = \sqrt{\frac{100 \sum_{N=1}^{100} (Y - f(X))^2}{100}} \quad (19)$$

where  $f(X)$  and  $Y$  are as defined in (2) and (18) respectively.

As an illustration, Table 7 demonstrates how the standard errors are obtained. Instead of substituting 100 input values as in this experiment, five equispaced inputs are used for simplicity. From Table 7 and equation (19), the standard error  $\sigma_{y.x}$  is thus equal to 0.298.

For further comparison, the standard error is commonly expressed in percent of full range, viz.,

$$\text{Standard Error in \%} = 100 \frac{\sigma_{y.x}}{R} = 100 \frac{1.7762}{1097.5} = 0.162 \quad (20)$$

Note that the fitted curve and its standard error in the series of the previous examples are only for a set of specific conditions: drawing ten samples (or  $M = 10$ ) at each of the five calibration steps (or  $N = 5$ ) for the specific input-output relationship,  $F(X) = 25 + 200 X + 0.75 X^2 + 0.30 X^3 + 0.05 X^4 + e_i$ , with precision  $\sigma_p = 0.6\%$ . If any one of these specific values is changed, a new fitted curve and a new standard error in percent is obtained. If all the levels under study are investigated, the total possible number of

TABLE 7

AN EXAMPLE FOR COMPUTING STANDARD ERROR OF A FITTED CURVE

WHEN  $N = 5$ 

X	Z	P <sub>0</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	Com. Val. From Y	Act. Val. From f(X)	Y-f(X)	(Y-f(X)) <sup>2</sup>
0.00	-2	1	-2	2	-1	24.586	25.000	-0.414	0.171
1.25	-1	1	-1	-1	2	274.799	276.880	-2.081	4.331
2.50	0	1	0	-2	0	534.286	536.328	-2.042	4.169
3.75	1	1	1	-1	-2	810.997	811.255	-0.258	0.067
5.00	2	1	2	2	1	1112.880	1112.500	0.380	0.144
Sum									8.882

fitted curves and total possible number of percentage standard error would be  $4860^7$ .

---

<sup>7</sup>This is obtained by the products of all possible levels, viz.,  $4 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 = 4860$ .

A computer simulation program was written to generate these 4860 values of percentage standard error. The sequence of steps utilized is the one previously outlined in this Chapter. The computer program is similar to the one shown in Appendix C of Technical Report Number 9, with some slight modifications to account for the different conditions.

Instead of showing in this thesis the lengthy computer program and corresponding printout, these have been filed for future reference in the Systems Engineering Group Laboratory, Bureau of Engineering Research, University of Alabama. However, a sample of the program's printout of values for the percentage standard error is exhibited in Table 8. Note that although the initial conditions remain unchanged, no value of the replications is identical to that obtained using equation (20). This can be attributed to the small probability of drawing identical random normal numbers.

TABLE 8

**A SAMPLE OF THE COMPUTER PROGRAM'S PRINTOUTS  
FOR THE PERCENTAGE STANDARD ERRORS**

$A_2$	$A_3$	$A_4$	$\sigma_p$	N	M	Replications				
0.00	0.00	0.00	.2	2	5	.033	.086	.078	.083	.062
0.00	0.00	0.00	.2	2	10	.030	.053	.022	.039	.036
0.00	0.00	0.00	.2	2	15	.064	.062	.025	.008	.040
.	.	.	.	.	.	.	.	.	.	.
0.75	0.30	0.05	.6	2	5	.217	.419	.205	.126	.093
0.75	0.30	0.05	.6	2	10	.124	.167	.154	.067	.090
0.75	0.30	0.05	.6	2	15	.245	.169	.117	.130	.138
0.75	0.30	0.05	.6	3	5	.177	.137	.192	.248	.202
0.75	0.30	0.05	.6	3	10	.189	.270	.041	.229	.377
0.75	0.30	0.05	.6	3	15	.108	.197	.121	.156	.053
0.75	0.30	0.05	.6	4	5	.380	.383	.357	.344	.268
0.75	0.30	0.05	.6	4	10	.151	.286	.202	.184	.163
0.75	0.30	0.05	.6	4	15	.146	.278	.318	.106	.099
0.75	0.30	0.05	.6	5	5	.266	.217	.241	.151	.307
*0.75	0.30	0.05	.6	5	10	.190	.228	.301	.270	.098*
0.75	0.30	0.05	.6	5	15	.287	.070	.195	.045	.116
0.75	0.30	0.05	1.0	2	5	.397	.422	.420	.226	.242
.	.	.	.	.	.	.	.	.	.	.
1.50	0.60	0.10	1.0	5	15	.584	.036	.166	.220	.212

\*This row has the same conditions as in equation (3).

## CHAPTER III

### ESTIMATION OF PARAMETERS

#### Factorial Experiment

It is important to recall that the terminal objective of this investigation is to determine the parameters, number of calibration levels and sample size at each level, for which the percentage standard error is a minimum. For this purpose, the errors associated with all levels of each factor are computed. Hence the foregoing experiment results in a factorial experiment with six factors (namely,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $S$ ,  $M$ , and  $N$ )<sup>8</sup> with five replications taken in each cell. Table 9 portrays the six-way factorial experiment in which each cell is a sum of five replications.

#### Assumptions

Prior to applying the technique of variance analysis, certain assumptions should be met (4):

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<sup>8</sup> $S$  denotes the precision  $\sigma$ , whereas  $A_2$ ,  $A_3$ ,  $A_4$ ,  $M$  and  $N$  share the same notation as in<sup>P</sup> Chapter II.

Table 9

THE NXMXSA<sub>2</sub>XA<sub>3</sub>XA<sub>4</sub> SIX-WAY FACTORIAL EXPERIMENT

			S = .2											
			N = 2			N = 3			N = 4			N = 5		
A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub> M	5	10	15	5	10	15	5	10	15	5	10	15
.00	.0	.00	0.342	0.180	0.199	0.250	0.212	0.154	0.244	0.191	0.161	0.150	0.222	0.225
		.05	0.233	0.307	0.182	0.278	0.306	0.121	0.360	0.343	0.253	0.423	0.267	0.290
		.10	0.416	0.129	0.213	0.325	0.322	0.219	0.490	0.246	0.369	0.366	0.271	0.321
	.3	.00	0.265	0.214	0.247	0.522	0.249	0.176	0.677	0.333	0.137	0.501	0.300	0.187
		.05	0.291	1.243	0.108	0.373	0.324	0.204	0.358	0.251	0.247	0.327	0.255	0.181
		.10	0.330	0.180	0.088	0.322	0.205	0.204	0.462	0.268	0.332	0.451	0.254	0.295
.75	.0	.00	0.222	0.171	0.185	0.309	0.345	0.212	0.310	0.335	0.260	0.329	0.227	0.263
		.05	0.427	0.183	0.183	0.403	0.324	0.177	0.396	0.199	0.197	0.320	0.290	0.277
		.10	0.315	0.252	0.169	0.323	0.290	0.221	0.384	0.229	0.180	0.335	0.430	0.288
	.3	.00	0.445	0.257	0.122	0.278	0.227	0.220	0.244	0.162	0.131	0.264	0.137	0.145
		.05	0.346	0.191	0.205	0.349	0.300	0.187	0.370	0.258	0.246	0.337	0.219	0.187
		.10	0.443	0.195	0.230	0.433	0.283	0.149	0.403	0.311	0.300	0.413	0.291	0.320
1.50	.6	.00	0.167	0.192	0.308	0.365	0.176	0.401	0.507	0.298	0.238	0.443	0.314	0.279
		.05	0.363	0.175	0.214	0.460	0.272	0.251	0.393	0.241	0.148	0.368	0.257	0.217
		.10	0.223	0.277	0.144	0.221	0.242	0.164	0.348	0.243	0.274	0.445	0.310	0.311
	.0	.05	0.178	0.225	0.156	0.241	0.296	0.175	0.405	0.264	0.235	0.400	0.238	0.144
		.10	0.305	0.188	0.204	0.451	0.314	0.252	0.349	0.272	0.217	0.267	0.254	0.317
		.15	0.383	0.237	0.155	0.334	0.244	0.284	1.148	0.303	0.255	0.336	0.292	0.220
	.3	.00	0.253	0.200	0.217	0.357	0.230	0.184	0.265	0.287	0.199	0.273	0.225	0.181
		.05	0.381	0.255	0.144	0.431	0.214	0.204	0.564	0.260	0.218	0.380	0.252	0.272
		.10	0.353	0.171	0.220	0.391	0.241	0.205	0.468	0.305	0.220	0.272	0.442	0.303
	.6	.00	0.244	0.236	0.153	0.266	0.265	0.185	0.483	0.412	0.188	0.502	0.217	0.190
		.05	0.229	0.250	0.231	0.375	0.336	0.188	0.260	0.386	0.762	0.346	0.187	0.247
		.10	0.398	0.240	0.126	0.288	0.258	0.160	0.321	0.216	0.177	0.480	0.305	0.289
	.0	.00	0.377	0.262	0.178	0.391	0.238	0.196	0.309	0.341	0.296	0.375	0.295	0.201
		.05	0.332	0.243	0.202	0.250	0.124	0.179	0.461	0.258	0.211	0.359	0.291	0.206
		.10	0.247	0.248	0.156	0.380	0.283	0.234	0.354	0.165	0.256	0.391	0.359	0.267

Table 9 (Continued)

A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> M				S = .6											
				N = 2			N = 3			N = 4			N = 5		
				5	10	15	5	10	15	5	10	15	5	10	15
.00	.0	.00	.00	0.586	0.784	0.505	1.079	0.718	0.474	0.847	0.555	0.518	0.886	0.429	0.493
			.05	1.096	0.616	0.378	1.009	0.903	0.580	1.256	1.311	0.928	1.107	1.311	0.915
			.10	1.278	0.652	0.715	0.937	0.763	0.773	1.386	0.906	0.603	1.733	0.753	0.544
	.3	.00	.00	0.797	0.816	0.547	1.199	0.807	0.645	1.093	0.939	0.886	1.022	0.724	0.803
			.05	1.140	0.488	0.317	1.008	1.244	0.541	1.730	1.339	0.698	1.121	1.098	0.961
			.10	0.854	0.823	0.433	1.030	0.517	0.373	1.298	0.801	0.678	1.312	0.668	0.603
.75	.0	.00	.00	1.555	0.596	0.713	0.961	1.001	0.545	1.534	1.134	0.959	1.179	0.821	0.949
			.05	0.739	0.733	0.649	0.972	0.998	0.719	1.316	1.017	0.982	0.966	0.713	1.490
			.10	1.080	0.716	0.619	1.230	0.878	0.489	1.995	0.775	0.869	1.799	0.937	0.517
	.3	.00	.00	0.774	0.691	0.482	1.071	1.256	0.767	0.318	0.819	0.759	1.161	0.918	0.833
			.05	0.773	0.714	0.415	0.652	0.668	0.900	1.442	1.093	0.776	1.203	1.112	0.844
			.10	1.093	0.686	0.679	0.944	0.647	0.667	1.508	1.349	0.568	0.997	1.296	0.459
1.50	.0	.00	.00	0.985	0.639	0.538	1.199	0.738	0.676	1.182	1.947	0.697	0.845	0.759	0.692
			.05	1.060	0.602	0.799	0.956	1.106	0.635	1.732	0.986	0.947	1.182	1.087	0.713
			.10	1.531	0.478	0.587	1.197	0.695	0.549	1.178	0.803	0.483	1.291	0.907	0.617
	.3	.00	.00	1.156	0.498	0.447	1.118	0.708	0.630	1.887	0.592	0.899	1.207	0.887	0.870
			.05	0.941	0.743	0.553	1.606	0.919	0.761	1.571	0.650	0.594	1.404	0.960	0.590
			.10	0.770	0.434	0.535	1.248	1.013	0.445	1.530	0.730	0.335	1.184	0.827	0.781
1.50	.0	.00	.00	1.254	0.776	0.535	1.196	0.608	0.839	1.316	1.242	0.859	1.488	1.101	0.741
			.05	1.158	0.853	0.646	1.196	0.865	0.546	1.648	0.960	0.782	1.575	0.733	0.816
			.10	1.091	0.694	0.471	0.897	0.865	0.585	1.145	0.920	0.942	0.748	0.765	0.755
	.3	.00	.00	0.980	0.630	0.478	0.984	0.658	0.585	1.145	0.920	0.942	0.748	0.765	0.755
			.05	1.176	0.700	0.500	0.795	0.774	0.507	1.568	1.054	0.717	1.589	0.691	0.814
			.10	1.444	0.750	0.476	1.192	0.504	0.687	1.647	1.335	0.487	1.210	0.751	0.550
1.50	.6	.00	.00	0.981	0.584	0.514	0.671	0.467	0.723	1.309	1.018	0.970	1.332	0.974	0.738
			.05	1.083	0.584	0.418	1.343	0.465	0.611	1.760	0.585	1.005	1.089	0.852	0.644
			.10	1.474	0.585	0.742	0.449	0.540	0.823	1.160	0.909	0.484	1.352	0.472	0.663

Table 9 (Continued)

A2			S = 1.0											
			N = 2			N = 3			N = 4			N = 5		
			A3	A4	M	5	10	15	5	10	15	5	10	15
.00	.0	.00	.00	.00	.00	0.955	1.092	1.022	1.440	0.943	1.122	0.298	0.776	0.721
		.05	.05	.05	.05	1.675	0.778	0.868	2.436	1.434	1.150	2.135	1.935	1.100
		.10	.10	.10	.10	1.988	0.798	0.810	0.471	1.227	0.990	2.688	1.790	1.612
	.3	.00	.00	.00	.00	1.588	1.083	1.077	3.023	1.025	1.275	2.521	1.361	0.782
		.05	.05	.05	.05	1.565	0.771	0.712	1.759	1.451	0.805	1.567	1.990	1.409
		.10	.10	.10	.10	1.263	1.397	0.904	0.943	1.244	0.735	2.518	2.199	1.920
.75	.6	.00	.00	.00	.00	1.241	0.673	0.888	2.114	1.116	1.006	2.032	1.760	1.489
		.05	.05	.05	.05	1.364	1.429	1.825	2.433	1.134	1.159	2.468	1.262	1.393
		.10	.10	.10	.10	1.348	0.892	1.016	1.812	1.824	1.194	3.046	1.516	1.025
	.0	.00	.00	.00	.00	1.346	1.415	0.754	1.614	1.735	1.180	1.804	1.367	0.958
		.05	.05	.05	.05	1.581	0.822	1.060	1.876	1.161	0.737	2.459	1.132	0.803
		.10	.10	.10	.10	1.999	1.157	0.654	2.205	1.017	1.004	2.454	1.806	1.643
1.50	.3	.00	.00	.00	.00	1.087	0.854	1.037	1.831	1.284	0.997	2.345	1.293	1.170
		.05	.05	.05	.05	1.707	1.854	0.866	2.401	0.997	1.114	2.155	1.193	1.249
		.10	.10	.10	.10	1.553	1.072	0.704	2.094	0.801	1.282	2.265	1.650	1.550
	.6	.00	.00	.00	.00	1.946	1.105	0.939	1.598	1.497	1.123	2.704	1.314	1.823
		.05	.05	.05	.05	1.793	0.760	1.398	2.142	1.219	1.141	2.216	1.730	1.595
		.10	.10	.10	.10	1.428	0.778	1.015	1.719	1.015	0.673	2.773	1.257	1.396
1.50	.0	.00	.00	.00	.00	1.184	0.761	0.590	2.029	1.086	0.813	1.951	1.027	2.012
		.05	.05	.05	.05	1.963	0.733	0.516	2.430	1.082	1.222	1.731	1.096	0.971
		.10	.10	.10	.10	1.019	1.092	1.140	1.744	1.279	0.981	2.070	1.663	1.225
	.3	.00	.00	.00	.00	1.142	1.314	1.404	2.107	1.157	1.461	1.176	1.229	0.940
		.05	.05	.05	.05	2.233	0.813	0.987	1.833	1.149	0.885	2.145	0.835	1.448
		.10	.10	.10	.10	1.707	1.117	0.742	1.788	1.326	1.193	2.053	1.686	1.183
1.50	.6	.00	.00	.00	.00	1.843	1.309	1.091	1.880	1.709	1.011	1.736	1.753	1.569
		.05	.05	.05	.05	1.273	1.129	1.016	1.747	1.451	1.007	2.526	1.282	1.525
		.10	.10	.10	.10	1.019	0.980	1.313	1.459	1.198	1.101	2.070	1.956	1.339
	.0	.00	.00	.00	.00	1.184	0.761	0.590	2.029	1.086	0.813	1.951	1.027	2.012
		.05	.05	.05	.05	1.963	0.733	0.516	2.430	1.082	1.222	1.731	1.096	0.971
		.10	.10	.10	.10	1.019	1.092	1.140	1.744	1.279	0.981	2.070	1.663	1.225

1. The sampled population is normally distributed.
2. The variance of the errors within all levels of each factor is homogeneous.
3. The experiment must be repeatable.

Since the experiment was simulated, it followed that these assumptions were naturally met. An additional assumption must be that all the effects are additive. In this analysis it is assumed that the simulated data met this requirement.

### Components

In the present case, a six-way analysis of variance with replications was used. The total variance can be partitioned into 63 components, viz.:

1. Main effects

$$C_1^6 = 6$$

2. First order interactions

$$C_2^6 = 15$$

3. Second order interactions

$$C_3^6 = 20$$

## 4. Third order interactions

$$C_4^6 = 15$$

## 5. Fourth order interactions

$$C_5^6 = 6$$

## 6. Fifth order interactions

$$C_6^6 = 1$$

These components represent all possible effects for the six-way factorial design. However, in this thesis only main effects, first order interactions and second order interactions were partitioned out while the higher order interactions were pooled with the random noise. It is fairly safe to do this since the variances of higher order interactions are relatively small as compared to the error variance and become even smaller as the order of the interactions increase (3,4, Table 10). Furthermore, even if these higher interactions were presented, they would be difficult to interpret in practical terms (4).

Mathematical Model

Hence, the mathematical model for the experimental design may be expressed as follows:

TABLE 10

## THE ANALYSIS OF VARIANCE OF THE SIMULATED DATA

Source of Variance	Sum of Squares	Degree of Freedom	Mean Square	F Ratio to Error Mean Square	Theoretical F Value at 1% level	
N	2.044	3	0.681	74.84	3.78	**
M	7.553	2	3.777	415.05	4.61	**
A <sub>2</sub>	0.015	2	0.008	0.87	4.61	
A <sub>3</sub>	0.135	2	0.068	7.47	4.61	**
A <sub>4</sub>	0.201	2	0.101	11.09	4.61	**
S	43.030	2	21.515	2364.69	4.61	**
N X M	0.097	6	0.016	1.76	2.80	
N X A <sub>2</sub>	0.031	6	0.005	0.54	2.80	
N X A <sub>3</sub>	0.092	6	0.015	1.65	2.80	
N X A <sub>4</sub>	0.283	6	0.047	5.16	2.80	**
N X S	0.792	6	0.132	14.50	2.80	**
M X A <sub>2</sub>	0.050	4	0.013	1.43	3.32	
M X A <sub>3</sub>	0.043	4	0.011	1.21	3.32	
M X A <sub>4</sub>	0.119	4	0.030	3.29	3.32	
M X S	2.217	4	0.554	60.87	3.32	**
A <sub>2</sub> X A <sub>3</sub>	0.044	4	0.011	1.21	3.32	
A <sub>2</sub> X A <sub>4</sub>	0.066	4	0.017	1.86	3.32	
A <sub>2</sub> X S	0.039	4	0.010	1.09	3.32	
A <sub>3</sub> X A <sub>4</sub>	0.155	4	0.039	4.29	3.32	**
A <sub>3</sub> X S	0.154	4	0.039	4.29	3.32	
A <sub>4</sub> X S	0.076	4	0.019	2.08	3.32	

\*\* Significant at 99% confidence level.

TABLE 10 (Continued)

Source of Variance	Sum of Squares	Degree of Freedom	Mean Square	F Ratio to Error Mean Square	Theoretical F Value of 1% level
N X M X A <sub>2</sub>	0.064	12	0.005	0.55	2.18
N X M X A <sub>3</sub>	0.194	12	0.916	1.75	2.18
N X M X A <sub>4</sub>	0.258	12	0.022	2.41	2.18 **
N X M X S	0.183	12	0.015	1.65	2.18
N X A <sub>2</sub> X A <sub>3</sub>	0.076	12	0.006	0.65	2.18
N X A <sub>2</sub> X A <sub>4</sub>	0.103	12	0.009	0.01	2.18
N X A <sub>2</sub> X S	0.146	12	0.012	1.31	2.18
N X A <sub>3</sub> X A <sub>4</sub>	0.243	12	0.020	2.11	2.18
N X A <sub>3</sub> X S	0.072	12	0.006	0.65	2.18
N X A <sub>4</sub> X S	0.431	12	0.036	3.95	2.18 **
M X A <sub>2</sub> X A <sub>3</sub>	0.102	8	0.013	1.43	2.51
M X A <sub>2</sub> X A <sub>4</sub>	0.099	8	0.012	1.31	2.51
M X A <sub>2</sub> X S	0.079	8	0.010	1.09	2.51
M X A <sub>3</sub> X A <sub>4</sub>	0.065	8	0.008	0.87	2.51
M X A <sub>3</sub> X S	0.073	8	0.009	0.10	2.51
M X A <sub>4</sub> X S	0.134	8	0.017	1.86	2.51
A <sub>2</sub> X A <sub>3</sub> X A <sub>4</sub>	0.076	8	0.010	1.09	2.51
A <sub>2</sub> X A <sub>3</sub> X S	0.078	8	0.010	1.09	2.51
A <sub>2</sub> X A <sub>4</sub> X S	0.092	8	0.012	1.31	2.51
A <sub>3</sub> X A <sub>4</sub> X S	0.119	8	0.015	1.65	2.51
Error	41.507	4576	0.0091		
Total	101.430	4859			

\*\* Significant at 99% confidence level.

Let  $N_i$ ,  $M_j$ ,  $(A_2)_k$ ,  $(A_3)_l$ ,  $(A_4)_m$  and  $S_n$  respectively denote various levels for main effects of the calibration levels, the sample size, the second-degree coefficient and the precision. Then the values of the response  $X_{ijklmnp}$  is:

$$\begin{aligned}
 X_{ijklmnp} = & U + N_i + M_j + (A_2)_k + (A_3)_l + (A_4)_m + S_n \\
 & + 15 \text{ first order interactions} \\
 & + 20 \text{ second order interactions} \\
 & + e_p (ijklmn)
 \end{aligned}$$

Now all possible effects through the second order interactions may be symbolically listed as follows:

1. Main effects:

$N \quad M \quad S \quad A_2 \quad A_3 \quad A_4$

2. First-order interaction effects:

$N \times M$	$N \times A_2$	$N \times A_3$	$N \times A_4$
$M \times A_2$	$M \times A_3$	$M \times A_4$	$S \times A_2$
$A_2 \times A_3$	$A_2 \times A_4$	$A_3 \times A_4$	$S \times A_3$
$A_2 \times S$	$A_3 \times S$	$A_4 \times S$	

### 3. Second-order interaction effects:

$N \times M \times A_2$	$N \times M \times A_3$	$N \times A_2 \times A_3$	$M \times A_2 \times A_3$
$N \times M \times A_4$	$N \times A_2 \times A_4$	$M \times A_2 \times A_4$	$N \times A_3 \times A_4$
$A_2 \times A_4 \times S$	$M \times A_3 \times A_4$	$N \times M \times S$	$N \times A_2 \times S$
$M \times A_3 \times S$	$N \times A_3 \times S$	$M \times A_3 \times S$	$A_2 \times A_3 \times S$
$N \times A_4 \times S$	$M \times A_4 \times S$	$A_3 \times A_4 \times S$	$A_2 \times A_3 \times A_4$

### Analysis of Variance

In order to compute the sum of squares for various sources, the data in Table 9 has been generated and cross-classified into some 48 computational tables. These tables might be computed by using a desk calculator. However in this thesis, most tables are obtained by a set of computer programs written in Fortran II, to save laborious work and to obtain the necessary accuracy. The programs and the cross-classification computational tables are also stored in the Systems Engineering Laboratory, Bureau of Engineering Research, University of Alabama.

After Table 9 has been properly classified, the table of analysis of variance (or ANOVA table) can be obtained as displayed in Table 10. Since the levels of all six factors

were assumed fixed, the F ratio for each source is obtained by dividing its respective mean square by the error mean square. Referring to Table X (7), the theoretical F values at the 99% confidence level may be found and are displayed in the last column of Table 10.

### Test of Significance

Table 10 indicated that each main effect except  $A_2$  is significantly different at the 99% confidence level. Of these five significant effects, the factor precision was the most significant one. The second most significant effect was due to M, the number of calibration levels. The third most significant effect was due to the various numbers of calibration levels N. As shown in Appendix B-1, the errors associated with two, three, four and five levels were 176.928, 200.719, 243.553 and 223.523 respectively. It is interesting that the least error was obtained when only two levels were used.

It was also found that  $A_3$  and  $A_4$  were statistically significant.

Of the interactions, five first order and two second order interactions were found to be moderately significant. These were:  $N \times A_4$ ,  $N \times S$ ,  $M \times S$ ,  $M \times S$ ,  $A_3 \times S$ ,  $N \times M \times A_4$  and  $N \times A_4 \times S$ .

### Components of Variance

One of the means to know the weight of each component in the total variance is to estimate the components of variance.

Table 11 which represented the Expectation of Mean Squares was established according to the Rule of EMS.<sup>9</sup> In order to determine the components of variance, the observed mean squares were set equal to the expected mean squares, and solved by means of a system of equations for the best estimates of the components. These expected variances are listed in the last column of Table 11. For the purpose of further comparisons, each expected variance was expressed in percentage of the total variance which is 0.027448 as presented in Table 12. The variance of precision, 48.36% of the total variance, revealed its tremendous effect on the whole system. Hence, any improvement in system precision may greatly assist in obtaining more accurate estimation.

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<sup>9</sup>EMS rule is a rule for setting up the table of Expected Mean Squares. The rule could be found in (4).

Table 11

## THE ANALYSIS OF COMPONENTS OF VARIANCE

Component	EMS Rule	Expected Mean Square		Expected Variance of the Component in Column 1
	F F F F F F R 4 3 3 3 3 3 5 N N A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> S e			
N	0 3 3 3 3 3 5	$\sigma_e^2 + 1215$	$\sigma_N^2 = .681$	.000553
M	4 0 3 3 3 3 5	$\sigma_e^2 + 1620$	$\sigma_M^2 = 3.777$	.002326
A <sub>2</sub>	4 3 0 3 3 3 5	$\sigma_e^2 + 1620$	$\sigma_{A_2}^2 = .008$	.000000
A <sub>3</sub>	4 3 3 0 3 3 5	$\sigma_e^2 + 1620$	$\sigma_{A_3}^2 = .068$	.000036
A <sub>4</sub>	4 3 3 3 0 3 5	$\sigma_e^2 + 1620$	$\sigma_{A_4}^2 = .101$	.000057
S	4 3 3 3 3 0 5	$\sigma_e^2 + 1620$	$\sigma_S^2 = 21.515$	.013275
NXM	0 0 3 3 3 3 5	$\sigma_e^2 + 405$	$\sigma_{NXM}^2 = .016$	.000017
NXA <sub>2</sub>	0 3 0 3 3 3 5	$\sigma_e^2 + 405$	$\sigma_{NXA_2}^2 = .005$	.000000
NXA <sub>3</sub>	0 3 3 0 3 3 5	$\sigma_e^2 + 405$	$\sigma_{NXA_3}^2 = .015$	.000015
NXA <sub>4</sub>	0 3 3 3 0 3 5	$\sigma_e^2 + 405$	$\sigma_{NXA_4}^2 = .047$	.000093
NXS	0 3 3 3 3 0 5	$\sigma_e^2 + 405$	$\sigma_{NXS}^2 = .132$	.000304
MXA <sub>2</sub>	4 0 0 3 3 3 5	$\sigma_e^2 + 540$	$\sigma_{MXA_2}^2 = .013$	.000007
MXA <sub>3</sub>	4 0 3 0 3 3 5	$\sigma_e^2 + 540$	$\sigma_{MXA_3}^2 = .011$	.000003
MXA <sub>4</sub>	4 0 3 3 0 3 5	$\sigma_e^2 + 540$	$\sigma_{MXA_4}^2 = .030$	.000039

Table 11 (Continued)

Component	EMS Rule							Expected Mean Square	Expected Variance of the Component in Column 1
	F	F	F	F	F	F	R		
	4	3	3	3	3	3	5		
	N	M	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	S	e		
MXS	4	0	3	3	3	3	5	$\sigma_e^2 + 540 \sigma_{MXS}^2 = .554$	.001009
A <sub>2</sub> XA <sub>3</sub>	4	3	0	0	3	3	5	$\sigma_e^2 + 540 \sigma_{A_2XA_3}^2 = .011$	.000003
A <sub>2</sub> XA <sub>4</sub>	4	3	0	3	0	3	5	$\sigma_e^2 + 540 \sigma_{A_2XA_4}^2 = .017$	.000011
A <sub>2</sub> XS	4	3	0	3	3	0	5	$\sigma_e^2 + 540 \sigma_{A_2XS}^2 = .010$	.000003
A <sub>3</sub> XA <sub>4</sub>	4	3	3	0	0	3	5	$\sigma_e^2 + 540 \sigma_{A_3XA_4}^2 = .039$	.000056
A <sub>3</sub> XS	4	3	3	0	3	0	5	$\sigma_e^2 + 540 \sigma_{A_3XS}^2 = .039$	.000056
A <sub>4</sub> XS	4	3	3	3	0	0	5	$\sigma_e^2 + 540 \sigma_{A_4XS}^2 = .019$	.000028
NXMxA <sub>2</sub>	0	0	0	3	3	3	5	$\sigma_e^2 + 135 \sigma_{NXMxA_2}^2 = .005$	.000000
NXMxA <sub>3</sub>	0	0	3	0	3	3	5	$\sigma_e^2 + 135 \sigma_{NXMxA_3}^2 = .016$	.000013
NXMxA <sub>4</sub>	0	0	3	3	0	3	5	$\sigma_e^2 + 135 \sigma_{NXMxA_4}^2 = .022$	.000096
NXMXS	0	0	3	3	3	0	5	$\sigma_e^2 + 135 \sigma_{NXMXS}^2 = .015$	.000044
NXA <sub>2</sub> XA <sub>3</sub>	0	3	0	0	3	3	5	$\sigma_e^2 + 135 \sigma_{NXA_2XA_3}^2 = .006$	.000000
NXA <sub>2</sub> XA <sub>4</sub>	0	3	3	3	0	3	5	$\sigma_e^2 + 135 \sigma_{NXA_2XA_4}^2 = .009$	.000000
NXA <sub>2</sub> XS	0	3	0	3	3	0	5	$\sigma_e^2 + 135 \sigma_{NXA_2XS}^2 = .012$	.000022

Table 11 (Continued)

Component	EMS Rule							Expected Mean Square	Expected Variance of the Component in Column 1
	F	F	F	F	F	F	R		
	4	3	3	3	3	3	5		
	N	M	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	S	e		
NXA <sub>3</sub> XA <sub>4</sub>	0	3	3	0	0	3	5	$\sigma_e^2 + 135 \sigma_{\text{NXA}_3\text{XA}_4}^2 = .020$	.000081
MXA <sub>3</sub> XS	4	0	3	0	3	0	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_3\text{XS}}^2 = .006$	.000000
MXA <sub>4</sub> XS	4	0	3	3	0	0	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_4\text{XS}}^2 = .036$	.000150
MXA <sub>2</sub> XA <sub>3</sub>	4	0	0	0	3	3	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_2\text{XA}_3}^2 = .013$	.000022
MXA <sub>2</sub> XA <sub>4</sub>	4	0	0	3	0	3	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_2\text{XA}_4}^2 = .012$	.000017
MXA <sub>2</sub> XS	4	0	0	3	3	0	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_2\text{XS}}^2 = .010$	.000006
MXA <sub>3</sub> XA <sub>4</sub>	4	0	3	0	0	3	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_3\text{XA}_4}^2 = .008$	.000000
MXA <sub>3</sub> XS	4	0	3	0	3	0	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_3\text{XS}}^2 = .009$	.000000
MXA <sub>4</sub> XS	4	0	3	3	0	0	5	$\sigma_e^2 + 180 \sigma_{\text{MXA}_4\text{XS}}^2 = .017$	.000044
A <sub>2</sub> XA <sub>3</sub> XA <sub>4</sub>	4	3	0	0	0	3	5	$\sigma_e^2 + 180 \sigma_{\text{A}_2\text{XA}_3\text{XA}_4}^2 = .010$	.000006
A <sub>2</sub> XA <sub>3</sub> XS	4	3	0	0	3	0	5	$\sigma_e^2 + 180 \sigma_{\text{A}_2\text{XA}_3\text{XS}}^2 = .010$	.000006
A <sub>2</sub> XA <sub>4</sub> XS	4	3	0	3	0	0	5	$\sigma_e^2 + 180 \sigma_{\text{A}_2\text{XA}_4\text{XS}}^2 = .012$	.000017
A <sub>3</sub> XA <sub>4</sub> XS	4	3	3	0	0	0	5	$\sigma_e^2 + 180 \sigma_{\text{A}_3\text{XA}_4\text{XS}}^2 = .015$	.000033
Residual	1	1	1	1	1	1	1	$\sigma_e^2 = .009$	.009000

Table 12

## COMPONENTS OF VARIANCE IN PERCENT OF TOTAL VARIANCE

Total Variance = 0.027448

Component of Variance	% of Total Variance	Component of Variance	% of Total Variance	Component of Variance	% of Total Variance
S	48.36	$NXA_3$	.05	$MXA_2XA_3$	.08
M	8.47	$A_2XA_4$	.04	$MXA_2XA_4$	.06
N	2.01	$MXA_2$	.02	$A_2XA_4XS$	.06
$A_4$	.21	$MXA_3$	.01	$NXMXA_3$	.04
$A_3$	.13	$A_2XA_3$	.01	$MXA_2XS$	.02
$A_2$	.00	$A_2XS$	.01	$A_2XA_3XA_4$	.02
MXS	3.67	$NXA_2$	.00	$MXNXA_2$	.00
NXS	1.10	$NXA_4XS$	.55	$NXA_2XA_4$	.00
$NXA_4$	.33	$NXMXA_4$	.34	$NXA_3XS$	.00
$A_3XA_4$	.20	$NXA_3XA_4$	.29	$A_2XA_3XS$	.00
$A_3XS$	.20	$NXMXS$	.16	$MXA_3XA_4$	.00
$MXA_4$	.14	$MXA_4XS$	.16	$MXA_3XS$	.00
$A_4XS$	.10	$A_3XA_4XS$	.12	$NXA_2XA_3$	.00
Residual	32.78	$NXM$	.06	$NXA_2XS$	.08

Next to the precision variability, the error variability accounts for 32.78% of the total variability. This error variability may result partly from the random errors and partly from pooling the third order and higher order interactions. Since these higher order interactions were small relative to the random error, a slight bias may result in the estimation of the error variance.

The sample size  $M$ , and number of calibration levels  $N$ , also contributed considerable effects, viz., 8.47% and 2.01% respectively. However, the coefficients  $A_2$ ,  $A_3$  and  $A_4$  contributed much less.

Most of the first order interactions indicated nonsignificant contributions except for  $MXS$  and  $NXS$  which contributed more than 1%. In Table 12, it can be seen that contributions by second-order interactions are very small. Many of them contributed less than 0.01%. It is very likely that interactions of higher order will even become much smaller.

#### The Significant Main Effects

The analysis of variance technique indicated that the main effects  $N$ ,  $M$ , and  $A_4$  are significant. However, the results do not indicate which of the individual levels of each factor were significantly different from one another. To obtain this goal, several alternative methods may be used (1). However, Duncan's Multiple Range Test will be employed

in this analysis. Since the test has the properties of larger Type I error and smaller Type II error than others, it is more likely to reject the null hypothesis and less likely to accept the alternative hypothesis. In testing hypothesis of the parameters for the telemetry system, we wish to use a technique which is sensitive to the detection of a small difference.

For those main effects of significance in the analysis, Duncan's Multiple Range Technique was further used. The necessary computations and procedures for various factors are illustrated in Appendix B, from B-1 through B-15. The results of the tests are summarized in Figure 2. The levels are ranked in ascending order. Any two means not underscored by the same line are significantly different and conversely.

In Figure 2.a, the results indicated that individual calibration levels are significantly different from one another with respect to grand means. However, the significant effect of N may be due to the wide variation for the different number of sample sizes. It is therefore desirable to investigate the values of N at sample sizes  $M = 5, 10$  and  $15$ . As exhibited in Figure 2.a, an increase in sample size averaged out the variation. For example, there was a significant difference from one level to the other when a sample of five at each level was taken, and there was no significance between 2 and 3 levels and also between 5 and 4

levels when a sample of 15 at each level was taken. It is also worth noting that there exists a significant least error when a calibration level  $N$  equals to two was used. This is true whenever the sample size is either 5, 10, or 15 points.

In Figure 2.b, the results revealed that different sample sizes at each level were significant from one another. However, the selection of sample sizes may cause a slight variability to the system's precision. When precision was selected at 0.2%, there was a significant difference between 5 and 10 samples, and there was not a significant difference between 10 and 15 samples. If the precision were reduced either to 0.6% or 1.0%, it would also become significantly different between 10 and 15 samples. Hence ten samples at each level may be adequate provided that the system's precision is assured equal to 0.2% or less; otherwise, sample size must be at least fifteen.

It would be desirable at this point to know how far the significant polynomial coefficients may vary while the present conclusions still hold true. For this purpose Duncan's Multiple Range Test was again used to test for  $A_3$  and  $A_4$ . The results are summarized in Figure 2.c and 2.d. For the grand mean of  $A_3$ , no significant difference was indicated between 0.0 and 0.3 nor between 0.3 and 0.6. For the grand mean of  $A_4$ , no significant difference was indicated between 0.00 and 0.10 nor between 0.10 and 0.05. If a sample of 5

or 10 is used at each level as shown in Figure 1.c and 1.d, there appears to be no significant difference among all three levels of  $A_3$  and  $A_4$ . This means that the coefficients of the third degree and the fourth degree may vary through 0.6 and 0.3 respectively, and will not affect the estimation of parameters provided that either ten or fifteen samples are used.

### The Significant Interaction Effects

In applying the general procedure of Duncan's Test the number of comparisons increases rapidly if the number of levels being compared becomes large. For instance, Table 17 in Appendix B would need  $11 + 10 + \dots + 2 + 1 = 66$  comparisons to perform Duncan's Multiple Range Test. In practice, a short-cut method (5) reduces this method to only ten comparisons. The first step is to subtract the least significant level associated with each mean from the largest observed mean. Any mean which is less than this difference is considered significant.

This concept of significance will always hold true since the Least Significant Range (or LSR) becomes smaller as a decrease in subset size occurs. This idea was repeatedly used in this short-cut in testing for significant interactions and the computations are presented in Appendix B. The summary of the computations is exhibited in Figure 3.

Figure 2

THE SUMMARIZED RESULTS OF DUNCAN'S MULTIPLE RANGE TESTS  
FOR THE SIGNIFICANT MAIN EFFECTS

2.a Tests With Respect To The Number Of Calibration Level N;

1. Grand Means:	<u>2</u>	<u>3</u>	<u>5</u>	<u>4</u>
2. Sample Size M = 5	<u>2</u>	<u>3</u>	<u>5</u>	<u>4</u>
3. Sample Size M = 10	<u>2</u>	<u>3</u>	<u>5</u>	<u>4</u>
4. Sample Size M = 15	<u>2</u>	<u>3</u>	<u>5</u>	<u>4</u>

2.b Tests With Respect To The Sample Size At Each Calibration Level M:

1. Grand Means:	<u>15</u>	<u>10</u>	<u>5</u>
2. Precision S = 0.2%:	<u>15</u>	<u>10</u>	<u>5</u>
3. Precision S = 0.6%:	<u>15</u>	<u>10</u>	<u>5</u>
4. Precision S = 1.0%:	<u>15</u>	<u>10</u>	<u>5</u>

2.c Tests With Respect To The Third-Degree Coefficient  $A_3$ ;

1. Grand Means:	<u>.0</u>	<u>.3</u>	<u>.6</u>
2. Sample Size M = 5:	<u>.0</u>	<u>.3</u>	<u>.6</u>
3. Sample Size M = 10	<u>.0</u>	<u>.3</u>	<u>.6</u>
4. Sample Size M = 15:	<u>.0</u>	<u>.3</u>	<u>.6</u>

Figure 2 (Continued)

2.d Tests With Respect To The Fourth-Degree Coefficient  $A_4$ :

1. Grand Means:	<u>.00</u>	<u>.10</u>	<u>.05</u>
2. Sample Size: M = 5	<u>.00</u>	<u>.05</u>	<u>.10</u>
3. Sample Size: M = 10	<u>.00</u>	<u>.05</u>	<u>.10</u>
4. Sample Size: M = 15	<u>.00</u>	<u>.05</u>	<u>.10</u>

Figure 3

THE SUMMARIZED RESULTS OF DUNCAN'S MULTIPLE RANGE TESTS  
FOR THE SIGNIFICANT FIRST-ORDER INTERACTION EFFECTS

3.a N X A<sub>4</sub> Interaction

(2;.00)\*(2;.10)(2;.05)(3;.10)(5;.00)(3;.00)(3;.05)(4;.00)(5;.05)(5;.10)(4;.05)(4;.10)

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3.b N X S Interaction

(2;.2)(3;.2) (5;.2) (4;.2) (2;.6) (3;.6) (5;.6) (4;.6) (2;.10) (3;.10) (5;.10) (4;.10)

\*Numbers in parentheses in this figure indicate the treatment combinations. The level of the first factor (N) is given first, then the level of the second factor (A<sub>4</sub> or S) is given. For instance, (2;.00) indicates that the treatment combination is of the level of 2 and .00 of the factors N and A<sub>4</sub> respectively.

Figure 3 (Continued)

3.c  $A_3 \times A_4$  Interaction

(.0;.00) (.3;.00) (.3;.10) (.0;.05) (.6;.10) (.0;.10) (.6;.00) (.3;.05) (.6;.05)

---

## 3.d M X S Interaction

(15;.2) (10;.2) (5;.2) (15;.6) (10;.6) (15;1.0) (5;.6) (10;1.0) (5;1.0)

---

3.e M X  $A_3$ 

(15;.0) (15;.3) (15;.6) (10;.0) (10;.3) (10;.0) (5;.0) (5;.3) (5;.6)

---

Studying the results of  $NXA_4$  interaction in Figure 3.a, it may be noted that those treatment combinations of same level of N and of different level of  $A_4$  tend to be nonsignificant. Table 12, components of variance, also showed that the effect of N in total variance was approximately ten times that of  $A_4$ . For these two reasons, a conclusion may be drawn that the significant difference among the  $MXA_4$  does not indicate that a considerable interaction effect between two factors is present.

Similarly, referring to Figures 3.b, 3.d and 3.e, those significant effects for interactions NXS, MXS and  $MXA_4$  could not be regarded as interaction effects. They resulted from the considerably different weights of two factors.

However, the weights for  $A_3$  and  $A_4$  were not considerably different. Hence, the significant error among the lowest levels of  $A_3XA_4$ , can be regarded as a significant interaction effect. For the other levels, no significant interaction effect can be recognized.

#### The Estimated Parameters

The optimal or sub-optimal estimates of the parameters of the simulated system can be made according to the results of a series of preceding tests. Two calibration levels provided the best fit to the present relationship between the input and output of the system. More specifically, a linear model, calibrated at zero volt and at five volts, is adequate

provided that at least 15 samples at each level are taken. It is very important to note that this statement is valid only if the linearity characteristics of the telemetry system prevail. That is, the results of the estimation of these parameters would be invalid if the conditions, viz.  $A_2$ ,  $A_3$ ,  $A_4$ ,  $Q_p$ , varied widely from those considered in the experiment.

In our simulated experiment, the linear characteristic is assumed existing in the system. This can be shown by considering the errors for all possible treatment combinations of the interaction  $A_2 \times A_3 \times A_4$ . The results of analysis of variance in Table 9 showed that the interaction was not significant. In other words, there was no significant difference among all treatment combinations of this interaction. Among these treatment combinations,  $A_2(0.0)A_3(0.0)A_4(0.0)$  symbolizes a linear relationship.<sup>10</sup> Thus we may conclude that the error produced by this line was not significantly different from that produced by other treatment combinations or curves. It follows that estimating the relationship of the specific system by a line is adequate at 95% confidence level.

---

<sup>10</sup> This denotes the treatment combination of the effects of  $A_2$ ,  $A_3$  and  $A_4$  respectively at the 0.0, 0.0, 0.0 levels.

Consider another case where the actual relationship is linear, symbolically,  $A_2=A_3=A_4=0.0$ . Duncan's Test was again performed over the various calibration levels, namely  $N = 2, 3, 4$  or  $5$ , for this linear model. It can be shown that the errors produced by various levels are not significantly different. Consequently, more calibration levels are not always necessarily required for fitting a curve to a set of observed data.

For another parameter,  $M$ , number of samples at each level, the error produced by sampling fifteen points was significantly less than produced by ten or five points. However, the previous analyses gave no indication of whether or not a value of  $M$  greater than 15 would continue to significantly reduce the error. If we can control the precision of the system as high as 0.2% of full range, then ten sample points at 0 and 5 volts each are satisfactory.

## CHAPTER IV

### CONCLUSIONS

#### Summary

This investigation has clarified the methodology presented in Technical Report Number 9, and expanded on some conditions considered in that report. The research and experimentation conducted throughout the course of this work have revealed several points including:

1. An experimental analysis of simulation data for a telemetry system has indicated that the standard five step calibration sequence is not the best one to determine the linearity characteristics of the system. Two calibration levels are adequate if at least fifteen values at each level can be obtained.

2. The analyses have also revealed that the precision or noise of the system significantly affects the determination of the true linearity characteristics of the system. If the precision is at less than or equal to 0.2% of full range can be controlled, ten values at 0 and 5 volts each are adequate.

3. Under the specific precisions considered, the coefficients of the polynomial output may vary through 1.50, 0.60 and 1.00 respectively for the second-degree, the third-degree and the fourth-degree. The optimal or suboptimal selections of calibration level and sample size will not be significantly affected by the variations of coefficients within these limits.

As the basis of the above results, a simulated system was developed reflecting the actual situations. The techniques of orthogonal regression analysis and variance analysis were employed in determining the parameters to be investigated.

#### Possible Sources of Error

The most likely source of error is the assumption of the output function in the form of a polynomial. The analyses throughout this thesis were based on this assumption. If other researches indicated that certain telemetry systems are truly non-polynomial, then the regression model presented in this thesis should be revised and the conclusions should be limited.

One possible source of errors in the analysis of variance could be due to the confounding of the third or higher degree interactions with the random error. Even though the possible error is nearly negligible, the inference from analysis of

variance may thus be affected.

Finally, the assumptions in regression analysis and the analysis of variance were accepted for this investigation. Significant departures from the assumptions would affect the inferences made.

### Recommendations for Future Research

In addition to the areas recommended in Technical Report Number 9, several areas of investigation are also open:

1. This investigation has shown that at least fifteen samples at each calibration level are required for the present simulated conditions. However, no conclusions can be drawn as to the possibility of requiring more than fifteen samples. Extensive research may be desirable considering 15, 20 or more samples.
2. The precision levels considered in this report were assumed to be between 0.2% and 1.0% of full range. In many FM/FM telemetry systems, however, precision measurements are frequently greater than 1%. For instance, the overall FM/FM mean precision of SA-8 telemetry data was 2.708% (6). Hence, some values around this mean may warrant further investigation.
3. This investigation has also shown that a two-step calibration sequence is adequate. Hence, it is recommended for an inflight experiment, using a two-

step calibration sequence, to be conducted in order to verify the findings reported herein.

4. Only the random noise was considered in this investigation. However, in an actual step-calibration sequence, there frequently exists an assignable effect which is called "damping" effect in the response (or output). This effect usually makes the signal oscillate acceleratedly toward the position of command signal. The time interval of this transient phenomenon varies with the nature of the system network and with each channel. Figure 4, showing the load position plotted against time, demonstrates a typical damping effect to the response of step command signal.

Since the damping effect, when it is large enough, will affect the determination of the parameters, namely, the number of calibration levels and the sample size at each level. Consequently, it is recommended to evaluate and quantify a mathematical expression for this effect.

Finally in order to comply with the unit of inputs used in previous reports, it is recommended for future simulation, that the range of inputs be changed from a 0 to 5 volts to a 0% to 100% range.

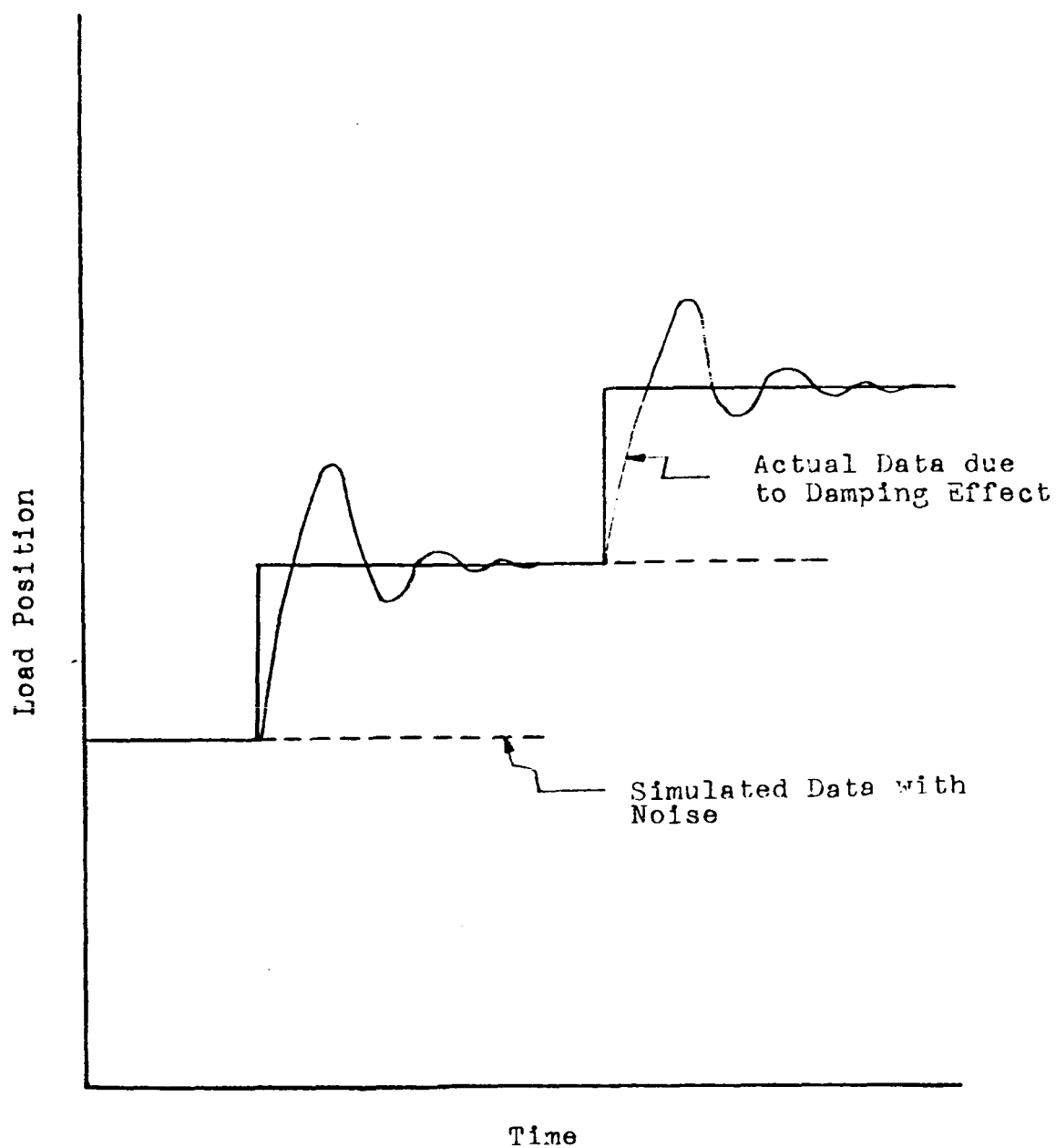


FIGURE 4 A CONCEPTUAL SEQUENCE OF SIGNAL RESPONSE TO STEP COMMAND WITH DAMPING EFFECT

**APPENDICES**

**APPENDIX A**

**GLOSSARY OF SYMBOLS**

## GLOSSARY OF SYMBOLS

- $A_0$  Intercept of the polynomial output.
- $A_1$  First degree coefficient of the polynomial output.
- $A_2$  Second degree coefficient of the polynomial output.
- $A_3$  Third degree coefficient of the polynomial output.
- $A_4$  Fourth degree coefficient of the polynomial output.
- $b$  Coefficients in the orthogonal polynomial output.
- $C$  Combination.
- $d.f.$  Degree of freedom.
- $e$  Random error.
- $f(X)$  True mathematical relationship.
- $F(X)$  Observed Mathematical relationship.
- $F(P)$  Orthogonal expression of  $F(X)$ .
- $F$  Value of the F-distribution.
- $i, j, k, l, m, n$  Indices of levels or summation.
- $M$  Number of samples at each calibration level.
- $N$  Number of calibration levels.
- $P$  Orthogonal polynomial.
- $R$  Range of a mathematical relationship.
- $\bar{R}$  Range of sample data.
- $S^2$  Sample Variance.
- $SS$  Sum of Squares associated with a given effect.
- $SS_M$  Sum of squares for mean.
- $\sigma^2$  Universe variance or unbiased estimate of a universe variance.
- $\sigma_a$  Accuracy of telemetry system in terms of standard deviation.

- $\sigma_e$  Standard deviation of random noise.
- $\sigma_p$  Precision of telemetry system in terms of standard deviation.
- $\sigma_{yx}$  Standard error of a fitting curve.
- $\mu$  Universe mean.
- $X$  Input value to the system in original unit.
- $X_{\max}$  Maximum input value in original unit.
- $X_{\min}$  Minimum input value in original unit.
- $Y$  Output value from the system.
- $\bar{Y}, \bar{Y}_i$  Mean output value at each calibration level.
- $\hat{Y}$  Least squares estimating equation.
- $Z$  Coded input value used in the orthogonal polynomial.

**APPENDIX B**

**COMPUTATION TABLES**

TABLE B-1

## DUNCAN'S MULTIPLE RANGE TEST AMONG THE NUMBER OF CALIBRATION LEVELS N WITH RESPECT TO GRAND MEANS

1. In the following table the values of the 'total' are obtained from Table 9 by summing up the correspondent cells and the values of the 'mean' are obtained by dividing the values of the 'total' by the number to be summed up.

N	2	3	4	5
Total	176.928	200.719	223.523	243.553
Mean	.146	.165	.184	.200

2. From Table 10 the error mean square,  $S_e^2 = .0091$  with degrees of freedom,  $df = 4576$ . Then the standard error of the mean is:  $S_{\bar{x}} = \sqrt{\frac{.0091}{1215}} = .00272$ .
3. From Table E (4), the significant range for  $n_2 = \infty$  is as follows (Since so far no available table as large as for  $df = 4576$ , the values for  $\infty$  is satisfactory).

P	2	3	4
Ranges	3.64	3.80	3.90

TABLE B-1 (Continued)

4. Multiplying by the standard error of .00272, the least significant ranges (LSR) are:

P	2	3	4
LSR	.010	.010	.011

5. Comparisons:

$.200 - .146 = .054 > .011$  \*\* \*\*significant at 1%  
 $.200 - .164 = .036 > .010$  \*\*  
 $.200 - .184 = .016 > .010$  \*\*  
 $.184 - .146 = .038 > .010$  \*\*  
 $.184 - .165 = .019 > .010$  \*\*  
 $.165 - .146 = .019 > .010$  \*\*

6. The results are exhibited in Figure 2.a.1.

TABLE B-2

DUNCAN'S MULTIPLE RANGE TEST AMONG THE NUMBER OF CALIBRATION  
LEVEL N WITH RESEPT TO M = 5

1.

N	2	3	5	4
Total	78.167	88.216	95.901	106.840
Mean	.193	.218	.238	.264

2. From Table 10,  $S_e^2 = .0091$  with  $df = 4576$   
Then, the standard error is:

$$S_{\bar{x}} = \sqrt{\frac{.0091}{405}} = .00474$$

3. From Table E (4), the significant ranges, for  
 $n_2 = \infty$  are:

P	2	3	4
Ranges	3.64	3.80	3.90

4. Multiplying by  $S_{\bar{x}}$ , the LSR's are:

P	2	3	4
LSR	.017	0.018	.019

5. Comparisons:

$$0.264 - 0.193 = 0.071 > 0.019 **$$

$$0.264 - 0.218 = 0.046 > 0.018 **$$

$$0.264 - 0.238 = 0.026 > 0.017 **$$

$$0.238 - 0.193 = 0.045 > 0.018 **$$

$$0.238 - 0.218 = 0.020 > 0.017 **$$

$$0.218 - 0.193 = 0.025 > 0.017 **$$

6. The results are exhibited in Figure 2.a.2.

TABLE B-3

DUNCAN'S MULTIPLE RANGE TEST AMONG THE NUMBER OF CALIBRATION  
LEVEL N WITH RESPECT TO M = 10

1.

N	2	3	5	4
Total	52.762	61.643	68.555	74.053
Mean	0.130	0.152	0.169	0.183

2,3,4 are same as Table B-2.

5. Comparisons

$$0.183 - 0.130 = 0.053 > 0.019 \quad **$$

$$0.183 - 0.152 = 0.031 > 0.018 \quad **$$

$$0.183 - 0.169 = 0.014 < 0.017$$

$$0.169 - 0.130 = 0.039 > 0.018 \quad **$$

$$0.169 - 0.152 = 0.017 = 0.017 \quad **$$

$$0.152 - 0.130 = 0.022 > 0.017 \quad **$$

6. The Results are exhibited in Figure 2.a.3.

TABLE B-4

DUNCAN'S MULTIPLE RANGE TEST AMONG THE NUMBER OF CALIBRATION  
LEVEL N WITH RESPECT TO M = 15

1.

N	2	3	5	4
Total	45.999	50.860	59.067	62.660
Mean	.114	.126	.146	.155

2,3,4 are the same as in Table B-2.

5. Comparisons :

$$.155 - .114 = .041 > .019 \quad **$$

$$.155 - .126 = .029 > .018 \quad **$$

$$.155 - .146 = .009 < .017$$

$$.146 - .114 = .032 > .018 \quad **$$

$$.146 - .126 = .020 > .017 \quad **$$

$$.126 - .114 = .012 < .017$$

6. The results are exhibited in Figure 2.a.d.

TABLE B-5

DUNCAN'S MULTIPLE RANGE TEST AMONG THE SAMPLE SIZE M WITH  
RESPECT TO GRAND MEANS

1.

M	15	10	5
Total	218.586	257.013	369.124
Mean	0.135	0.159	0.228

$$2. \quad S_e^2 = 0.0091, \quad df = 4576$$

$$S_e = \sqrt{0.0091/1620} = .00237$$

3. From Table E (4), the significant ranges, for  $n_2 = \infty$  are:

P	2	3
Ranges	3.64	3.80

4. Multiplying by  $S_g$ , the LSR's are:

P	2	3
LSR	.009	.009

5. Comparisons:

$$0.228 - 0.135 = 0.093 > 0.009 \quad **$$

$$0.228 - 0.159 = 0.069 > 0.009 \quad **$$

$$0.159 - 0.135 = 0.024 > 0.009 \quad **$$

6. The results are exhibited in Figure 2.b.1.

TABLE B-6

DUNCAN'S MULTIPLE RANGE TEST AMONG THE SAMPLE OF SIZE M WITH  
RESPECT TO PRECISION  $S = 0.2$

1.

M	15	10	5
Total	23.835	28.817	39.060
Mean	0.044	0.053	0.072

2.  $S_e^2 = 0.0091$ , d.f. = 4576

$$S_{\bar{x}} = \sqrt{\frac{0.0091}{540}} = .004$$

3. Same as Table B-5

4.

P	2	3
LSR	.015	.016

5. Comparisons:

$$0.072 - 0.044 = 0.028 > 0.016 \quad **$$

$$0.072 - 0.053 = 0.019 > 0.015 \quad **$$

$$0.053 - 0.044 = 0.009 < 0.015$$

6. The results are exhibited in Figure 2.b.2.

TABLE B-7

DUNCAN'S MULTIPLE RANGE TEST AMONG THE SAMPLE SIZE M WITH  
RESPECT TO PRECISION  $S = 0.6$

1.

M	15.	10	5
Total	71.601	88.839	127.647
Mean	0.133	0.165	0.236

2,3,4 are same as Table B-6

5. Comparisons

$$0.236 - 0.133 = 0.103 > 0.016 \quad **$$

$$0.236 - 0.165 = 0.071 > 0.015 \quad **$$

$$0.165 - 0.133 = 0.032 > 0.015 \quad **$$

6. The results are exhibited in Figure 2.b.3.

TABLE - 8

DUNCAN'S MULTIPLE RANGE TEST AMONG THE SAMPLE SIZE M WITH  
RESPECT TO PRECISION  $S = 1.0$

1.

M	15	10	5
Total	123.150	139.357	202.417
Mean	0.228	0.258	0.375

2,3,4 are same as Table B-6

5. Comparisons:

$$0.375 - 0.228 = 0.147 > 0.016 **$$

$$0.375 - 0.258 = 0.117 > 0.015 **$$

$$0.258 - 0.228 = 0.030 > 0.015 **$$

6. The results are exhibited in Figure 2.b.4.

TABLE B - 9

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_3$  COEFFICIENTS WITH  
RESPECT TO GRAND MEANS

1.

$A_3$	0.0	0.3	0.6
Total	271.469	280.943	292.311
Mean	0.168	0.173	0.180

2,3,4 are same as Table B-5

5. Comparisons:

$$0.180 - 0.168 = 0.012 > 0.009 **$$

$$0.180 - 0.173 = 0.007 < 0.009$$

$$0.173 - 0.168 = 0.005 < 0.009$$

6. The results are exhibited in Figure 2.c.1.

TABLE B-10  
DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_3$  COEFFICIENTS WITH  
RESPECT TO  $M = 5$

1.

$A_3$	0.0	0.3	0.6
Total	117.799	122.443	128.882
Mean	0.218	0.227	0.239

2,3,4 are same as Table B-6

5. Comparisons:

$$\begin{array}{rclcl}
 0.239 - 0.218 & = & 0.021 & 0.016 & ** \\
 0.239 - 0.227 & = & 0.012 & 0.015 & \\
 0.227 - 0.218 & = & 0.009 & 0.015 & 
 \end{array}$$

6. The results are exhibited in Figure 2.c.2.

TABLE B-11

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_3$  COEFFICIENTS WITH  
RESPECT TO  $M = 10$

1.

$A_3$	0.0	0.3	0.6
Total	84.486	86.212	86.315
Mean	0.156	0.160	0.160

2,3,4 are same as Table B - 6

5. Comparisons:

$$0.160 - 0.156 = 0.004 < 0.016$$

$$0.160 - 0.160 = 0.000 < 0.015$$

$$0.160 - 0.156 = 0.004 < 0.015$$

6. The results are exhibited in Figure 2.c.3

TABLE B-12

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_3$  COEFFICIENTS WITH  
RESPECT TO  $M = 15$

1.

$A_3$	0.0	0.3	0.6
Total	69.184	72.288	77.114
Mean	0.128	0.134	0.143

2,3,4 are same as Table B-6

5. Comparisons:

$$0.143 - 0.128 = 0.015 < 0.016$$

$$0.143 - 0.134 = 0.009 < 0.015$$

$$0.134 - 0.128 = 0.006 < 0.015$$

6. The results are exhibited in Figure 2.c.4.

TABLE B-13

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_4$  COEFFICIENTS WITH  
RESPECT TO GRAND MEANS  $A_4$

1.

$A_4$	0.00	0.10	0.05
Total	267.003	287.060	290.660
Mean	0.165	0.177	0.179

2,3,4 are same as Table B-5

5. Comparisons:

$$\begin{aligned}
 0.179 - 0.165 &= 0.014 > 0.009 & ** \\
 0.179 - 0.177 &= 0.002 < 0.009 \\
 0.177 - 0.165 &= 0.012 > 0.009 & **
 \end{aligned}$$

6. The results are exhibited in Figure 2.d.1.

TABLE B-14

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_4$  COEFFICIENTS WITH  
RESPECT TO  $M=5$

1.

$A_4$	0.0	0.05	0.10
Total	113.305	126.885	128.934
Mean	0.210	0.235	0.239

2,3,4 are same as Table B-6

5. Comparisons:

$$\begin{aligned}
 0.239 - 0.210 &= 0.029 > 0.016 & ** \\
 0.239 - 0.235 &= 0.004 < 0.015 \\
 0.235 - 0.210 &= 0.025 > 0.015 & **
 \end{aligned}$$

6. The results are exhibited in Figure 2.d.2.

TABLE B-15

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_4$  COEFFICIENTS WITH  
RESPECT TO M=10

1.

$A_4$	0.0	0.10	0.05
Total	82.627	86.455	87.931
Mean	0.153	0.160	0.163

2,3,4 are same as Table B-6

5. Comparisons:

$$0.163 - 0.153 = 0.010 < 0.016$$

$$0.163 - 0.160 = 0.003 < 0.015$$

$$0.160 - 0.153 = 0.007 < 0.015$$

6. The results are exhibited in Figure 2.d.3.

TABLE B-16

DUNCAN'S MULTIPLE RANGE TEST AMONG THE  $A_4$  COEFFICIENTS WITH  
RESPECT TO M=15

1.

$A_4$	0.0	0.10	0.05
Total	71.071	71.671	75.844
Mean	0.132	0.133	0.140

2,3,4 are same as Table B-6

5. Comparisons:

$$0.140 - 0.132 = 0.008 < 0.016$$

$$0.140 - 0.133 = 0.007 < 0.015$$

$$0.133 - 0.132 = 0.001 < 0.015$$

6. The results are exhibited in Figure 2.d.4.

TABLE-17

DUNCAN'S MULTIPLE RANGE TEST AMONG ALL LEVELS OF THE  $N \times A_4$  INTERACTION BY THE SHORT-CUT

## METHOD

Column Treat- ment Combi- nation	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	$(2;.00)(2;.10)(2;.20)(3;.10)(3;.20)(3;.30)(3;.40)(3;.50)(4;.00)(4;.05)(4;.10)(4;.15)(4;.20)(4;.25)(4;.30)(4;.35)(4;.40)(4;.45)(4;.50)(5;.00)(5;.05)(5;.10)(5;.15)(5;.20)(5;.25)(5;.30)(5;.35)(5;.40)(5;.45)(5;.50)(6;.00)(6;.05)(6;.10)(6;.15)(6;.20)(6;.25)(6;.30)(6;.35)(6;.40)(6;.45)(6;.50)(7;.00)(7;.05)(7;.10)(7;.15)(7;.20)(7;.25)(7;.30)(7;.35)(7;.40)(7;.45)(7;.50)(8;.00)(8;.05)(8;.10)(8;.15)(8;.20)(8;.25)(8;.30)(8;.35)(8;.40)(8;.45)(8;.50)(9;.00)(9;.05)(9;.10)(9;.15)(9;.20)(9;.25)(9;.30)(9;.35)(9;.40)(9;.45)(9;.50)(10;.00)(10;.05)(10;.10)(10;.15)(10;.20)(10;.25)(10;.30)(10;.35)(10;.40)(10;.45)(10;.50)(11;.00)(11;.05)(11;.10)(11;.15)(11;.20)(11;.25)(11;.30)(11;.35)(11;.40)(11;.45)(11;.50)(12;.00)(12;.05)(12;.10)(12;.15)(12;.20)(12;.25)(12;.30)(12;.35)(12;.40)(12;.45)(12;.50)$											
Total	56.934	59.120	60.611	62.611	67.334	68.551	69.547	74.174	77.649	78.540	82.590	86.789
Mean	.141	.146	.150	.155	.166	.169	.172	.183	.192	.194	.204	.214

\*Number in parenthesis in above table indicate the treatment combinations. The level of the first factor N is given first, then the level of the second factor  $A_4$ .

P	2	3	4	5	6	7	8	9	10	11	12
Ranges	3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17	4.20	4.23	4.26
LSR	.017	.018	.019	.019	.019	.019	.019	.020	.020	.020	.020

1. Compare all means with Col. (12).

A.  $0.214 - 0.020 = 0.194$

Subtract LSR of Col. (12) from its mean. Since the means from Col. (1) through Col. (9) are less than 0.194, it is concluded that these means are

TABLE B-17 (Cont'd)

significantly different from Col. (12). However, Col. (10) and Col. (11) need more comparisons.

B.  $0.214 - 0.018 = 0.196$

Subtract LSR of Col. (3) from mean of Col. (12). Since the mean of Col. (10) less than this difference and the mean of Col. (11) greater than this, it is concluded that the former is significantly different from Col. (12) and the latter is not yet decided.

C.  $0.214 - 0.017 = 0.197$

Subtract LSR of Col. (2) from mean of Col. (12). Since the mean of Col. (11) is greater than this difference, it is not significantly different from Col. (12).

The results of step 1 can be summarized as all means except  $N_{(4)A(4)}(.05)$  are significantly different from  $N_{(4)A(4)}(.10)$ . These are denoted by drawing a line underscoring of Col. (11) and (12).

## 2. Compare with Col. (11).

A.  $0.204 - 0.020 = 0.184$

Then Col. (1) through Col. (8) are significantly different from Col. (11), Col. (9) and Col. (10) Need further comparisons.

B.  $0.204 - 0.018 = 0.186$   
 $0.204 - 0.017 = 0.187$

Col. (9) and Col. (10) are not significantly different from Col. (11).

## 3. Compare with Col. (10)

A.  $0.194 - 0.020 = 0.174$

Col. (1) through (7) are significantly different from Col. (10).

B.  $0.194 - 0.018 = 0.176$

Col. (8) and Col. (9) are not

C.  $0.194 - 0.017 = 0.177$

significantly different from Col. (10).

TABLE B-17 (Cont'd)

4. Compare with Col. (9)		Col. (1) through (7) are significantly different Col. (8) is not significantly different from Col. (9).
A.	$0.192 - 0.020 = 0.172$	
B.	$0.192 - 0.017 = 0.175$	
5. Compare with Col. (8)		Col. (1) through (4) are significantly different from Col. (8).
A.	$0.183 - 0.020 = 0.163$	
B.	$0.183 - 0.019 = 0.164$	Col. (5) through (7) are not significantly different from Col. (8).
6. Compare with Col. (7)		Col. (1) through (3) are significantly different from Col. (7).
A.	$0.172 - 0.019 = 0.153$	
B.	$0.172 - 0.019 = 0.153$	Col. (4) through (6) are not.
7. Compare with Col. (6)		Col. (1) through (2) are significantly different from Col. (6).
A.	$0.169 - 0.019 = 0.150$	
B.	$0.169 - 0.018 = 0.151$	Col. (3) are too.
C.	$0.169 - 0.018 = 0.151$	Col. (4) and Col. (5) are not.
8. Compare with Col. (5)		Col. (1) and Col. (2) are significantly different from Col. (5)
A.	$0.166 - 0.189 = 0.147$	
B.	$0.166 - 0.180 = 0.151$	Col. (3) is significant too.
C.	$0.166 - 0.174 = 0.148$	Col. (4) is not.

TABLE B-17 (Cont'd)

9. Compare with Col. (4)  
 $0.155 - 0.019 = 0.136$  Col. (1) through (3) are not significantly different from Col. (4).

The result of these comparisons is summarized in Fig. 3.a.

TABLE B-18

DUNCAN'S MULTIPLE RANGE TEST AMONG ALL LEVELS OF THE NXS INTERACTION BY THE SHORT-CUT

METHOD

Column Treat- ment combi- nation	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	(2;.2)(3;.2)(5;.2)(4;.2)(2;.6)(3;.6)(5;.6)(4;1.0)(2;1.0)(3;1.0)(5;.1.0)(4;1.0)											
Total	20.366	22.052	23.877	25.417	61.426	65.817	76.304	84.540	95.136	112.850	123.342	133.596
Mean	.050	.055	.059	.063	.152	.163	.188	.209	.279	.305	.330	.330

The table of LSR is exactly the same as in Table B-16.

1. Compare all means with Col. (12)

$$0.330 - 0.020 = 0.310$$

All means are significantly different from Col.(12).

TABLE B-18 (Cont'd)

2. Compare with Col. (11)	Col. (1) through Col. (10) are significantly different from Col. (11)
0.305 - 0.020 = 0.285	
3. Compare with Col. (10)	Col. (1) through Col. (9) are significantly different from Col. (10).
0.279 - 0.020 = 0.259	
4. Compare with Col. (9)	Col. (1) through Col. (8) are significantly different from Col. (9).
0.235 - 0.020 = 0.215	
5. Compare with Col. (8)	Col. (1) through Col. (7) are significantly different from Col. (8).
0.209 - 0.020 = 0.189	
6. Compare with Col. (7)	Col. (1) through Col. (6) are significantly different from Col. (7).
0.188 - 0.019 = 0.169	
7. Compare with Col. (6)	Col. (1) through Col. (4) are significantly different from Col. (6).
0.163 - 0.019 = 0.144	
0.163 - 0.017 = 0.146	Col. (5) are not.

TABLE B-18 (Cont'd)

8. Compare with Col. (5)  
0.152 - 0.019 = 0.133  
Col. (1) through Col. (4) are significantly different from Col. (5).
9. Compare with Col. (4)  
0.063 - 0.019 = 0.044  
Col. (1) through Col. (3) are not significantly different from Col. (4).

The result of these comparisons is summarized in Fig. 3.b.

TABLE B-19

DUNCAN'S MULTIPLE RANGE TEST AMONG ALL LEVELS OF THE  $A_3 \times A_4$  INTERACTION BY THE SHORT-CUT METHOD

Column Treatment Combination	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(.0;.00)(.3;.00)(.3;.10)(.0;.05)(.6;.10)(.0;.10)(.6;.00)(.3;.05)(.6;.05)								
Total	80.456	89.470	93.892	94.080	96.235	96.933	97.077	97.581	98.999
Mean	.149	.166	.174	.174	.178	.179	.180	.181	.183

TABLE B-19 (Cont'd.)

P	2	3	4	5	6	7	8	9
Ranges	3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17
LSR	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.017

## 1. Compare with Col. (9)

- A.  $0.183 - 0.017 = 0.166$  Col. (1) and (2) are significantly different from Col. (9).  
 B.  $0.183 - 0.015 = 0.168$  Col. (3) through (8) are not.

## 2. Compare with Col. (8)

- A.  $0.181 - 0.017 = 0.164$  Col. (1) is significantly different from Col. (8).  
 B.  $0.181 - 0.015 = 0.166$  Col. (2) through (7) are not.

## 3. Compare with Col. (7)

- A.  $0.180 - 0.017 = 0.163$  Col. (1) is significantly different from Col. (7).  
 B.  $0.180 - 0.015 = 0.165$  Col. (2) through (6) are not.

## 4. Compare with Col. (6)

- A.  $0.179 - 0.017 = 0.162$  Col. (1) is significantly different from Col. (6).  
 B.  $0.179 - 0.015 = 0.164$  Col. (2) through (5) are not.

TABLE B-19 (Cont'd)

5. Compare with Col. (5)
  - A.  $0.178 - 0.016 = 0.162$  Col. (1) is significantly different from Col. (5).
  - B.  $0.178 - 0.015 = 0.163$  Col. (2) through (4) are not.
6. Compare with Col. (4)
  - A.  $0.174 - 0.016 = 0.158$  Col. (1) is significantly different from Col. (4).
  - B.  $0.174 - 0.015 = 0.159$  Col. (2) and Col. (3) are not
7. Compare with Col. (3)
  - A.  $0.174 - 0.016 = 0.158$  Col. (1) is significantly different from Col. (3).
  - B.  $0.174 - 0.016 = 0.168$  Col. (2) is not.
8. Compare with Col. (2)
  - $0.166 - 0.015 = 0.151$  Col. (1) is significantly different from Col. (2)

The result of these comparisons is summarized in Fig. 3.c.

### DUNCAN'S MULTIPLE RANGE TEST AMONG ALL LEVELS OF THE MXS INTERACTIONS BY THE SHORT-

## CUT METHOD

Column Treatment Combination	(1) (15;.2)	(2) (10;.2)	(3) (5;.2)	(4) (15;.6)	(5) (10;.6)	(6) (15;1.0)	(7) (5;.6)	(8) (10;1.0)	(9) (5;1.0)
Total	23.835	28.817	39.060	71.601	88.839	123.150	127.647	139.357	202.417
Mean	.0444	.053	.072	.133	.165	.228	.237	.258	.375

The table of LSR is exactly the same as in Table B-18.

- |                          |  |   |
|--------------------------|--|---|
| 1. Compare with Col. (9) | 0.375 - 0.017 = 0.358                                | Col. (1) through (8) are significantly different from Col. (9).                 |
| 2. Compare with Col. (8) | 0.258 - 0.017 = 0.241                                | Col. (1) through (7) are significantly different from Col. (8).                 |
| 3. Compare with Col. (7) | A. 0.237 - 0.017 = 0.220<br>B. 0.237 - 0.015 = 0.222 | Col. (1) through (5) are significantly different from (7).<br>Col. (6) are not. |

TABLE B-20 (Cont'd)

4. Compare with Col. (6)	
0.228 - 0.017 = 0.211	Col. (1) through (5) are significantly different from (16).
5. Compare with Col. (5)	
0.165 - 0.016 = 0.149	Col. (1) through (4) are significantly different from (5).
6. Compare with Col. (4)	
0.133 - 0.016 = 0.117	Col. (1) through (3) are significantly different from (4).
7. Compare with Col. (3)	
0.072 - 0.016 = 0.056	Col. (1) are significantly different from (3).
8. Compare with (2)	
0.053 - 0.015 = 0.038	Col. (1) is not significantly different from (2).

The result of these comparisons is summarized in Fib. 3.d.

TABLE B-21  
DUNCAN'S MULTIPLE RANGE TEST AMONG ALL LEVELS OF THE MXA<sub>3</sub> INTERACTION BY THE SHORT-CUT METHOD

Column Treatment Combination	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(15;.0)(15;.3)(15;.6)(10;1.0)(10;.3)(10;.6)(5;.0)(5;.3)(5;.6)								
Total	69.184	72.288	77.114	84.486	86.212	86.315	117.799	122.443	128.882
Mean	.128	.134	.143	.157	.160	.160	.218	.227	.239

The table of LSR is exactly the same as in Table C-3.

TABLE B-21 (Cont'd)

1. Compare with Col. (9)			
A. $0.239 - 0.017 = 0.222$	Col. (1) through (7) are significantly different from (9).		
B. $0.239 - 0.015 = 0.224$	Col. (8) is not.		
2. Compare with Col. (8)			
A. $0.181 - 0.017 = 0.164$	Col. (1) through (6) are significantly different from (8).		
B. $0.181 - 0.015 = 0.166$	Col. (7) is not.		
3. Compare with Col. (7)			
$0.218 - 0.017 = 0.201$	Col. (1) through (6) are significantly different from (7).		
4. Compare with Col. (6)			
A. $0.160 - 0.017 = 0.143$	Col. (1) through (3) are significantly different from (6).		
B. $0.160 - 0.015 = 0.145$	Col. (4) and (5) are not.		
5. Compare with Col. (5)			
A. $0.160 - 0.016 = 0.144$	Col. (1) through (3) are significantly different from (5).		
B. $0.160 - 0.015 = 0.145$	Col. (4) is not.		
6. Compare with Col. (4)			
$0.157 - 0.016 = 0.141$	Col. (1) through (2) are significantly different from (4).		
$0.157 - 0.015 = 0.142$	Col. (3) is not.		

TABLE B-21 (Cont'd)

7. Compare with Col. (3)

- |    |                         |   |
|----|-------------------------|---|
| A. | $0.143 - 0.016 = 0.127$ | Col. (1) is significantly different from (3). |
| B. | $0.143 - 0.015 = 0.128$ | Col. (2) is not.                              |

8. Compare with Col. (2)

- |                         |  |
|-------------------------|--|
| $0.134 - 0.015 = 0.119$ | Col (1) is not significantly different from (2). |
|-------------------------|--|

The result of these comparisons is summarized in Fig. 3.e.

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